In the example of doubling rice each day (see video) we had a "doubling time" of one day and an initial amount of 1 grain of rice.

<u>Terms</u>: $f(t) = y_0 e^{tr}$

initial amount

base of exponential (often e, but can be other numbers as well)

growth rate

Generalize $y_0 b^{rt}$

| Amount of rice | Days |
|---|---|
| 1 | 0 |
| 2 | 1 |
| $2^2 = 4$ | 2 |
| $2^3 = 8$ | 3 |
| $2^4 = 16$ | 4 |
| $2^5 = 32$ | 5 |
| $2^6 = 64$ | 6 |
| $2^7 = 128$ | ~~8~~7 |

→ Pattern

On the $t^{th}$ day, we need $2^t$ grains of rice ← exponential

Ex: Consider $f(t) = 2^{t}$

Write in form

$$g(t) = y_0 e^{rt}$$

Trick: $2^{t} = e^{\ln(2^{t})}$  $e^{a+b} = e^{a}e^{b}$

$\ln(2) \approx 0.693$

$$= e^{\ln(2)t}$$

$$= 1 e^{0.693 t}$$

initial amount of 1

growth rate of 0.693

How does doubling time relate to growth rate?

$(r>0)$

$$f(t) = y_0 e^{rt}$$

← at time $t=0$, initial pop of $y_0$

At what value of $t$ will the $\underbrace{\text{population double}}_{2y_0}$?

Solve for $t$:

$$2\cancel{y_0} = \cancel{y_0} e^{rt} \rightarrow 2 = e^{rt}$$

double the pop.

$$\rightarrow \ln(2) = rt$$

$$\rightarrow \boxed{t = \frac{\ln(2)}{r}}$$

Ex: Suppose $f(t)=\boxed{3}e^{\boxed{0.01}t}$

r

What is the doubling time?

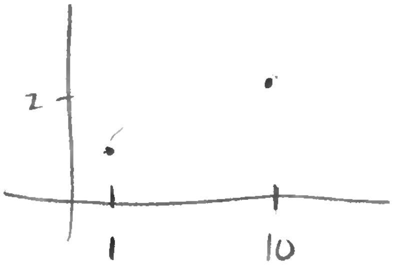
initial

$$6=3e^{0.01t}$$

$$2=e^{0.01t}\longrightarrow \frac{\ln(2)}{0.01}=t\approx 69.31$$

Ex: $f(t)=6000e^{\boxed{0.01}t}$

What is doubling time?



$$12000=6000e^{0.01t}$$

$$2=e^{0.01t}$$

$$\rightarrow t\approx\boxed{69.31}$$

Ex: $f(t)=\boxed{200}e^{0.005t}$

What is doubling time?

$$400=200e^{0.005t}$$

$$2=e^{0.005t}\longrightarrow t=\frac{\ln(2)}{0.005}\approx 138.62$$

Ex: Given a doubling time of $T=30$ what is growth rate?

$$f(t)=y_0 e^{rt}$$

$$2\cancel{y_0} = f(30) = \cancel{y_0} e^{30r}$$

$$\ln(2)=30r$$

$$\boxed{r=\frac{\ln(2)}{30}}$$

We arrived at two formulas:

$$\boxed{\begin{gathered}\text{doubling time} = \frac{\ln(2)}{\text{growth rate}} \\ \text{growth rate} = \frac{\ln(2)}{\text{doubling time}}\end{gathered}}$$