$(a^b)^c = a^{bc}$

Ex: $49^{3x} = 7$

one way

$\log_{49}(49^{3x}) = \log_{49}(7)$

$3x = \log_{49}(7)$

$x = \frac{\log_{49}(7)}{3}$

$\sqrt{49} = 7$

$7 = 49^{1/2}$

$= \frac{\log_{49}(49^{1/2})}{3}$

$= \frac{1}{6}$

2nd way

$\ln(49^{3x}) = \ln(7)$

$3x \ln(49) = \ln(7)$

a number

$x = \frac{\ln(7)}{3\ln(49)}$

$= \frac{\ln(7)}{3\ln(7^2)}$

$= \frac{1}{6}$

3rd

Since

$49 = 7^2$

can write equation as

$(7^2)^{3x} = 7$

$7^{6x} = 7$

$\log_7(7^{6x}) = \log_7(7^1)$

$6x = 1$

$x = \frac{1}{6}$

Ex: Solve

$\log_x 8 = \frac{1}{9}$

↓ make both side exponent of x

$x^{\log_x(8)} = x^{\frac{1}{9}}$

$8 = x^{\frac{1}{9}}$

↓ take both sides to 9th power

$8^9 = (x^{\frac{1}{9}})^9$

↓

$\boxed{8^9 = x}$

$\log_a(a^x) = x$

$a^{\log_a x} = x$

Ex: Solve

$$e^{x+3} = e^x + 1$$

TRICKY

~~Natural~~ to try: ln both sides

$$\ln(e^{x+3}) = \ln(e^x + 1)$$

$$x+3 =$$

can't use inverse property!! ~ the "+1" gets in the way

Solution 1

Recall: $e^{a+b} = e^a e^b$

So, $e^{x+3} = e^x e^3$

$$e^x e^3 = e^x + 1$$

$$e^x e^3 - e^x = 1$$

$$e^x(e^3 - 1) = 1$$

$e^3 - 1$: a number

$$e^x = \frac{1}{e^3 - 1}$$

ln

$$\ln(e^x) = \ln\left(\frac{1}{e^3-1}\right)$$

$\ln(1) - \ln(e^3-1)$, $\ln(1) = 0$

$$x = -\ln(e^3 - 1)$$

Soln 2

Multiply both sides by e^{-x}

$$e^{-x} e^{x+3} = e^{-x} e^x + e^{-x}$$

$$e^{-x+x+3} = e^{-x+x} + e^{-x}$$

$$e^3 = e^0 + e^{-x}$$

$e^0 = 1$

$$e^3 - 1 = e^{-x}$$

take ln

$$\ln(e^3 - 1) = \ln(e^{-x})$$

$$\ln(e^3 - 1) = -x$$

mult by -1

$$x = -\ln(e^3 - 1)$$

Ex: Solve $\ln(x^4) = (\ln(x))^2$ (can't simplify)

$4\ln(x) = (\ln(x))^2$

$0 = (\ln(x))^2 - 4\ln(x)$

$0 = \ln(x)[\ln(x) - 4]$

zero prod prop

$a \cdot b = 0$

$a = 0$ or $b = 0$

Could think of this as letting

$w = \ln(x)$

$4w = w^2$

$0 = w^2 - 4w$

$0 = w(w-4)$

OR

$\ln(x) = 0$

plug into e^x

$e^{\ln(x)} = e^0$

$x = 1$

$\ln(x) - 4 = 0$

$\ln(x) = 4$

$e^{\ln(x)} = e^4$

$x = e^4$

Ex: Use calculator to solve

$$3^{-x} = 2x - 5$$

since there is an x in an exponential then x outside of one, we can't solve using our usual tricks!

$$x \approx 2.531$$

Ex: "Lambert W function"

Consider $f(x) = xe^x$