Solve logarithmic + exponential equations

$\log_{18}(x)$ is inverse of 18^x

$\cancel{\log_{18}(18^x)} = x = \cancel{18}^{\log_{18}(x)}$

Ex: Solve for x:

$$\log_{18}(3x+2) = \log_{18}(x+1)$$

$\downarrow$ plug both sides into 18^x

$$\cancel{18}^{\log_{18}(3x+2)} = \cancel{18}^{\log_{18}(x+1)}$$

$\downarrow$ inverse property

$$3x+2 = x+1 \longrightarrow 2x = -1$$

$$\boxed{x = -1/2}$$

Ex: $\log_7(2x) - 3 = \log_7(x-1)$

Recall

$$a^{b+c} = a^b a^c$$

$\downarrow$ plug both sides into 7^x

$$7^{\log_7(2x)-3} = \cancel{7}^{\log_7(x-1)}$$

$$\left(\cancel{7}^{\log_7(2x)}\right)\left(7^{-3}\right) = x-1$$

$$7^{-3} = \frac{1}{7^3}$$

$$7^{-3}(2x) = x-1$$

$$\frac{2}{7^3}x = x-1$$

$$\underbrace{\left(\frac{2}{7^3}-1\right)}_{\text{a number}}x = -1$$

$$x = \frac{-1}{\frac{2}{7^3}-1} \approx 1.01$$

$\ln(a) - \ln(a) = \ln\left(\frac{a}{b}\right)$

Ex: $\ln(x+1) - \ln(x+7) = 4$

ln is $\log_e$

$\ln\left(\frac{x+1}{x+7}\right) = 4$

inverse of $\ln(x)$ is e^x

Technically this has no soln!

plug into e^x

$\cancel{e}^{\cancel{\ln}\left(\frac{x+1}{x+7}\right)} = e^4$

inverse prop

$\frac{x+1}{x+7} = e^4$

This one has soln $x \approx -7.112$

mult by $(x+7)$

$x+1 = e^4(x+7)$

$e \approx 2.71$

$x+1 = e^4 x + 7e^4$

number

$x - e^4 x = 7e^4 - 1$

$x\underbrace{(1-e^4)}_{\text{number}} = 7e^4 - 1$

$x = \frac{7e^4 - 1}{1 - e^4} \approx -7.112$

Cannot plug this into original equation b/c we get things like $\ln(-7.112 + 1) = \ln(-6.112)$ NOT DEF

Ex: $5^{x+1} = 12$

Two SOLNS

Use fact that

$$\log_5(5^{x+1}) = x+1$$

(inverse property)

↓ plug both sides of original eqt into $\log_5(x)$

$$\cancel{\log_5}(5^{x+1}) = \log_5(12)$$

↓

$$x+1 = \log_5(12)$$

$$\boxed{x = \log_5(12) - 1} \approx 0.543$$

same

↓ weird 3rd way

$$\log_2(5^{x+1}) = \log_2(12)$$

$$(x+1)\log_2(5) = \log_2(12)$$

$$x = \frac{\log_2(12)}{\log_2(5)} - 1$$

same

Use ln ~ exploit property that $\ln(x^{a}) = a\ln(x)$

↓ plug original eqt into $\ln(x)$ function

$$\ln(5^{x+1}) = \ln(12)$$

$$(x+1)\ln(5) = \ln(12)$$

$$x+1 = \frac{\ln(12)}{\ln(5)}$$

$$\boxed{x = \frac{\ln(12)}{\ln(5)} - 1} \approx 0.543$$

same

Same

Change of base formula: converts $\log_a$ to $\log_b$

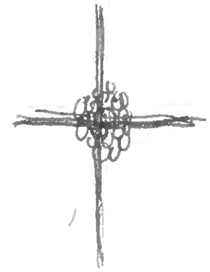
$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

Ex: $e^x = 12$

$\big\downarrow$ plug into ln

$\cancel{\ln}(\cancel{e}^x) = \ln(12)$

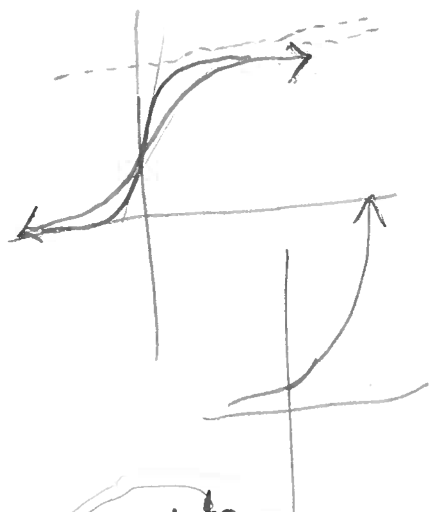
$x = \ln(12)$



Ex: $\ln(\ln(\ln(3x))) = 6$

$\big\downarrow$ plug into e^x

$\cancel{e}^{\cancel{\ln}(\ln(\ln(3x)))} = e^6$



$\big\downarrow$

$\ln(\ln(3x)) = e^6$

related to Gompertz growth models (~1825)

$\big\downarrow$ plug into e^x

$\cancel{e}^{\cancel{\ln}(\ln(3x))} = e^{e^6}$

$\ln(3x) = e^{e^6}$

$\big\downarrow$ plug into e^x

$\cancel{e}^{\cancel{\ln}(3x)} = e^{e^{e^6}} \rightarrow 3x = e^{e^{e^6}}$

$x = \frac{1}{3}e^{e^{e^6}}$

Ex: Solve

$$2(3^x+1) = 6-3(3-3^x)$$

$$2\cdot 3^x + 2 = \underbrace{6-9}_{-3} + 3\cdot 3^x$$

$$2+3 = 3^x$$

$$5 = 3^x$$

plug into $\log_3(x)$

$$\boxed{x = \log_3(5)}$$

plug into ln

$$\ln(5) = x\ln(3)$$

$$\boxed{x = \frac{\ln(5)}{\ln(3)}}$$

10^2

$\log_{10}(100)$

$= 2$