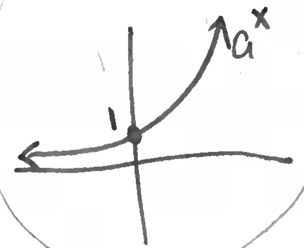
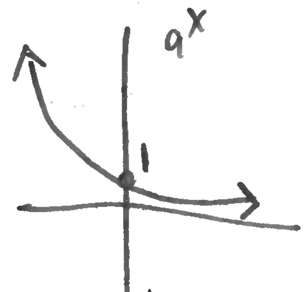
# Logarithms

**FACT:** if $a>0$



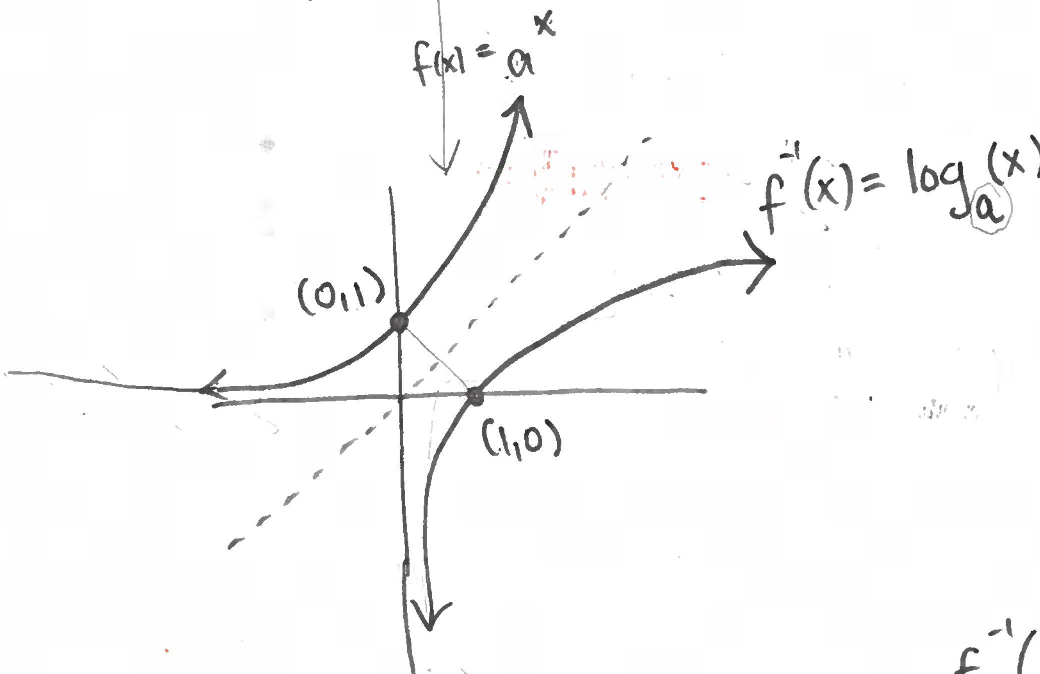

if $0<a<1$




Bothe of these are one-to-one functions

Therefore, we can find inverse function,

$\ln(ab) = \ln(a) + \ln(b)$




**inverse property:** $f(f^{-1}(x)) = x$ $\qquad f^{-1}(f(x)) = x$

$$a^{\log_a(x)} = x \qquad \log_a(a^x) = x$$

Particularly important: $a = e$ ← special number $e \approx 2.71\ldots$

$\log_e(x)$ is so important we call it the "natural logarithm"

and we use special notation: $\ln(x)$ — $e^{\ln(x)} = x$

$$\ln(e^x) = x$$

<u>EX</u>: Find a formula for $\underbrace{\text{the exponential}}_{\text{usually mean take base to be } e}$ that satisfies

$$f(0)=6 \text{ and } f(1)=18.$$

two values

<u>Soln</u>: Generally: $\boxed{f(x) = e^{Ax+B}}$

two unknowns (A, B)

Plug in given info:

$$\underset{\text{given}}{6} = f(0) = \underset{\text{calculated}}{e^{A \cdot 0 + B}}$$

$\boxed{6=e^{B}}$ ← WANT B

↓ plug into ln

$\ln(6) = \cancel{\ln(e}^{B})$

$\boxed{B=\ln(6)}$

$$\underset{\text{given}}{18} = f(1) = \underset{\text{compute}}{e^{A(1)+\ln(6)}}$$

$18 = e^{A+\ln(6)} \xrightarrow[\text{to ln}]{\text{plug in}} \ln(18) = \cancel{\ln}(\cancel{e}^{A+\ln(6)})$

$\ln(18) = A + \ln(6)$

$A = \ln(18) - \ln(6)$

The answer

$$f(x) = e^{(\ln(18)-\ln(6))x + \ln(6)}$$

3

## Other properties of logs

$\log_a(xy) = \log_a(x) + \log_a(y)$

"log of product is sum of logs"

$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

"log of quotient is difference of logs"

Ex: $\log_a(x^2) = \log_a(x \cdot x) = \log_a(x) + \log_a(x)$

$= 2\log_a(x)$

Ex: $\log_a(x^3) = \log_a(x \cdot x^2) = \log_a(x) + \log_a(x^2)$

$= \log_a(x) + 2\log_a(x)$

$= 3\log_a(x)$

Generally: $\log_a(x^b) = b\log_a(x)$

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log

"means $\log_{10}$" - might be true to some

"means $\log_2$" ~ CS

"means ln" - all math people