1

fact: $(a-b)(a+b) = (a-b)a + (a-b)b$

$= a^2 - \cancel{ba} + \cancel{ba} - b^2 = a^2 - b^2$

$(x-2)(x+2) = x^2 - 4$

Ex: Solve

$-1 + \frac{1}{3} - \frac{2}{-3}$

$-1 + \frac{1}{3} + \frac{2}{3} = 0$ ✓

$$\underbrace{\frac{1}{x-2} + \frac{1}{x+2}}_{\text{common denom is } x^2-4} - \frac{2}{x^2-4} = 0$$

$$\frac{x+2}{x^2-4} + \frac{x-2}{x^2-4} - \frac{2}{x^2-4} = 0$$

$$\frac{x+2+(x-2)-2}{x^2-4} = 0$$

does not contribute to soln

$\sim$ $x \neq \pm 2$

but otherwise multiply it away

$$2x - 2 = 0$$

$$2x = 2$$

$$\boxed{x = 1}$$

Exponentials and logarithms

so far we've mostly looked at polynomials

e.g. $2x^3 - 3x^2 + x - 1$

Shape of polynomials: $x^{\text{fixed number}}$

fixed exponent

variable base

exponential

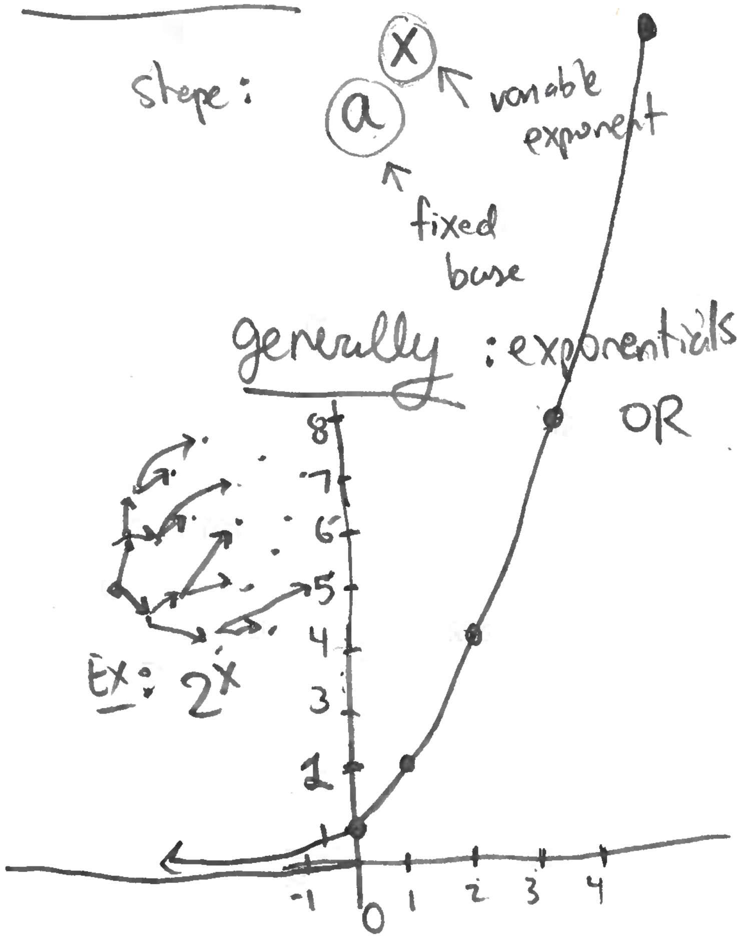
Shape: a^x

variable exponent

fixed base

generally: exponentials grow really fast OR decay really fast

Ex: 2^x

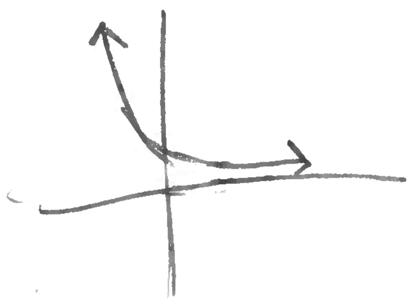


x	2^x
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$
-1	$2^{-1} = \frac{1}{2}$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

Ex:

x	$\left(\frac{1}{10}\right)^x$
0	1
1	$1/10$
2	$(1/10)^2 = (1/10)(1/10) = \frac{1}{100}$
-1	$(1/10)^{-1} = \frac{1/1}{1/10} = \left(\frac{1}{1}\right)\left(\frac{10}{1}\right) = 10$
-2	$(1/10)^{-2} = \frac{1}{1/100} = 100$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right)$$



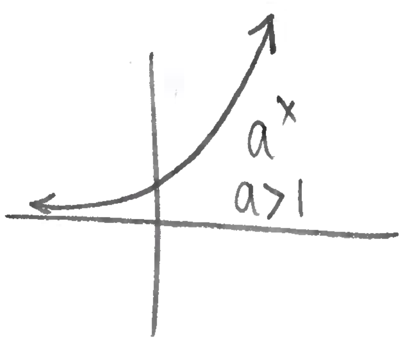
Units

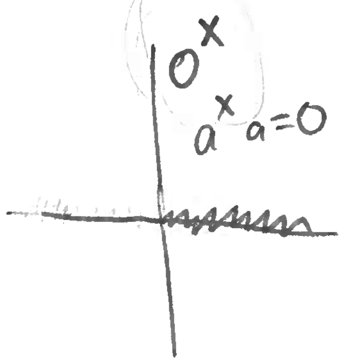
Chem ~ pH

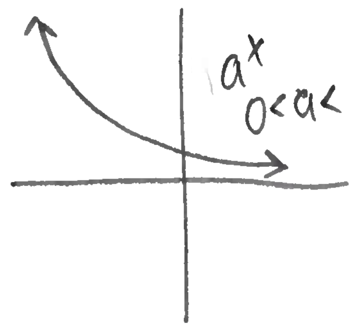
decibel ~

magnitude equake

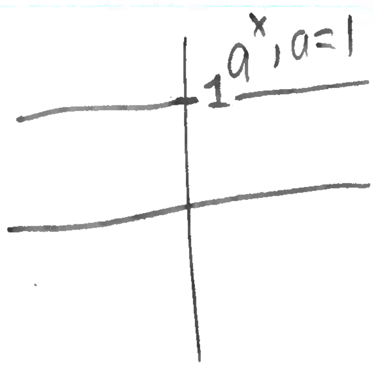
Four possible shapes for graph of exponentials (for non-neg base)







$\cancel{-1^{1/2} = i}$



Ex: $2^3 2^2 = \underbrace{(2\cdot 2\cdot 2)}_{2^3}\underbrace{(2\cdot 2)}_{2^2} = 2\cdot 2\cdot 2\cdot 2\cdot 2 = 2^5$

$$2^3 2^2 = 2^{3+2} = 2^5$$

$$x^2 x^3 = x^{2+3} = x^5$$

$$2^{1/2} 2^{1/3} = 2^{\frac{1}{2}+\frac{1}{3}} = 2^{\frac{5}{6}}$$

$\frac{1}{2} = \frac{3}{6}$

$\frac{1}{3} = \frac{2}{6}$

$$\boxed{a^b a^c = a^{b+c}}$$

Ex: $2^{3x+1} 2^{x-5} = 2^{3x+1+x-5} = 2^{4x-4}$

The number $e \approx 2.71\ldots$

Like π, the number e is a famous irrational number.

"base of the "natural" exponential"

$$\frac{d}{dx} e^x = e^x$$

In general if e appears do NOT replace w/ 2.71...

Turns out that $e, \pi, i = \sqrt{-1}$ are related by a simple formula

$$e^{\frac{i\pi}{2}} = i$$

$$i^i = e^{-\frac{\pi}{2}}$$

$$\boxed{e^{i\pi} + 1 = 0} \leftarrow \text{Euler's identity}$$

$$e^{i\theta} = \cos(\theta) + i\sin(\theta) \leftarrow \text{Euler's}$$