Ex: $\dfrac{\sqrt{5-x^2}}{1} - \dfrac{3}{\sqrt{5-x^2}} = 1$

$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$

Soln: Common denominator of left-hand side is $\sqrt{5-x^2}$.

$$\frac{\left(\sqrt{5-x^2}\right)^2 - 3}{\sqrt{5-x^2}} = 1$$

$$\frac{5-x^2-3}{\sqrt{5-x^2}} = 1$$

$(-x^2+2)^2 = (-x^2+2)(-x^2+2)$
$= x^4 - 4x^2 + 4$

↓ mult by $\sqrt{5-x^2}$

$$-x^2+2 = \sqrt{5-x^2}$$

↓ square

$$(-x^2+2)^2 = \left(\sqrt{5-x^2}\right)^2$$

$$x^4 - 4x^2 + 4 = 5 - x^2$$

(*) $x^4 - 3x^2 - 1 = 0$ ← "quadratic in form"

Introduce a new variable: $w = x^2$

$w^2 = (x^2)^2 = x^4$

In w, (*) becomes

$$w^2 - 3w - 1 = 0 \xrightarrow{QF} w = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2}$$

$$= \frac{3 \pm \sqrt{13}}{2}$$

Since

$$\begin{cases} x^2 = w = \dfrac{3+\sqrt{13}}{2} \longrightarrow x = \pm\sqrt{\dfrac{3+\sqrt{13}}{2}} = \pm 1.82 \\ x^2 = w = \dfrac{3-\sqrt{13}}{2} \longrightarrow x = \pm\sqrt{\dfrac{3-\sqrt{13}}{2}} = \pm 0.55i \end{cases}$$

then check

Ex: $\sqrt[3]{3x+5} + 5 = 0$

$(\sqrt{x})^2 = x$

$(\sqrt[3]{x})^3 = x$

$\sqrt[3]{3x+5} = -5$

↓ cube both sides

$3x+5 = (-5)^3$

$3x+5 = -125$

$3x = -130$

$x = -\dfrac{130}{3}$