Ex: Product of two consecutive even integers is 728. Find those integers.

{..., -2, -1, 0, 1, 2, ...}

{..., -2, 0, 2, 4, 6, ...}

next to each other

Soln: Let x be one of the two even integers.

The next consecutive even integer is $x+2$.

So,

$ax^2+bx+c=0$

$$x(x+2)=728$$

$$x^2+2x-728=0$$

$\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$ ↓ QF

$$x=\frac{-2\pm\sqrt{2^2-4(1)(-728)}}{2}$$

$$=\frac{-2\pm\sqrt{4+2912}}{2}$$

$$=-1\pm\frac{1}{2}\sqrt{2916}$$

$$=-1\pm\frac{54}{2}=-1\pm 27$$

$+$ → 26 ; $-$ → -28

use $x=26 \rightarrow x+2=28$

use $x=-28 \rightarrow x+2=-26$

2 —(+2)→ 4 —(+2)→ 6

2: next consec. even integer

4: next consec. even int

6

Rational equations and radical equations

means that we have fractions with polynomials (in x) on top & on bottom

square roots, cube roots, etc with x inside

Radical eqts

Ex: $\sqrt{x-1} = \sqrt{2x-3}$

recall $(\sqrt{\ast})^{\ast} \cdot x$ inverses!

square both sides

$(\sqrt{x-1})^{2} = (\sqrt{2x-3})^{2}$

inverse property

$x-1 = 2x-3$

$2 = x$

check

$\sqrt{2-1} \stackrel{?}{=} \sqrt{2(2)-3}$

$\sqrt{1} = \sqrt{1}$ ✓

BEWARE: When solving radical eqts, MUST ALWAYS check solns ~ i.e. plug back in to original

Ex: $\sqrt{x} = x - 5$

↓ square 2

$x = (x-5)^2$

$x = x^2 - 10x + 25$

$0 = x^2 - 11x + 25$ $\quad 121 - 100$

↓ QF

$$x = \frac{-(-11) \pm \sqrt{121 - 4(1)(25)}}{2}$$

$$= \frac{11}{2} \pm \frac{1}{2}\sqrt{21}$$

(+) / (−)

$x = \frac{11}{2} + \frac{1}{2}\sqrt{21}$ $\qquad$ $x = \frac{11}{2} - \frac{1}{2}\sqrt{21}$

↓ Check $\qquad$ ↘ Check

$\sqrt{\frac{11}{2} + \frac{1}{2}\sqrt{21}} \stackrel{?}{=} \frac{11}{2} + \frac{1}{2}\sqrt{21} - 5$ $\qquad$ $\sqrt{\frac{11}{2} - \frac{\sqrt{21}}{2}} \stackrel{?}{=} \left(\frac{11}{2} - \frac{\sqrt{21}}{2}\right) - 5$

↓ ↓ $\qquad$ $1.791 \stackrel{?}{=} -1.791$

$2.791 = 2.791$ ✓ $\qquad$ NO

↑

only soln

Ex: $\sqrt{x} = \sqrt{-1-x}$

↓ square

$x = -1-x$

↓

$2x = -1$

↓

$x = -\frac{1}{2}$

$a, b > 0$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

check

$\sqrt{-\frac{1}{2}} = \sqrt{-1-(-\frac{1}{2})}$

$-\frac{1}{2}$

$\frac{1}{\sqrt{2}}i = \frac{1}{\sqrt{2}}i$ ✓

Ex: $\sqrt{x-5} - \sqrt{x} = 1$

$\sqrt{x-5} = 1 + \sqrt{x}$

↓ square

$x-5 = (1+\sqrt{x})^2$

isolate + square again

$x-5 = 1+2\sqrt{x}+x$

$-5 = 1+2\sqrt{x}$

$-6 = 2\sqrt{x}$

$-3 = \sqrt{x}$

↓ square

$9 = x$

check

$\sqrt{9-5} - \sqrt{9} \stackrel{?}{=} 1$

$\sqrt{4} - \sqrt{9} \stackrel{?}{=} 1$

$2 - 3 \stackrel{?}{=} 1$ NO

NO SOLN