$x^2 + bx = \left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$ ①

$x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

Ex: $2x^2 - x + 1 < 2$

problem for completing square

must be divided off

divide both sides by 2

$\frac{a+b+c}{d} = \frac{a}{d} + \frac{b}{d} + \frac{c}{d}$

$$\frac{\cancel{2}x^2 - x + 1}{\cancel{2}} < \frac{2}{2}$$

$$\frac{\cancel{2}x^2}{\cancel{2}} - \frac{x}{2} + \frac{1}{2} < 1$$

$$\boxed{x^2 + \underbrace{\left(-\tfrac{1}{2}\right)}_{\text{"b"}}x} < \frac{1}{2}$$

$\frac{1}{2} + \frac{1}{16} = \frac{8}{16} + \frac{1}{16}$

complete square

$b = -\frac{1}{2}$

$\frac{b}{2} = -\frac{1}{4}$

$\left(\frac{b}{2}\right)^2 = \left(-\frac{1}{4}\right)^2$

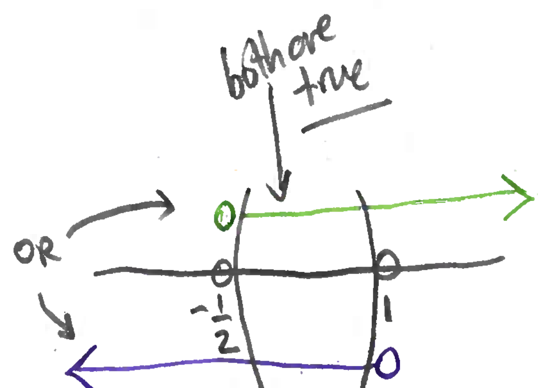
$= \frac{1}{16}$

$$\left(x - \tfrac{1}{4}\right)^2 - \left(\tfrac{1}{16}\right) < \frac{1}{2}$$

$$\left(x - \tfrac{1}{4}\right)^2 < \frac{9}{16}$$

take $\sqrt{\ }$

$$\pm\left(x - \tfrac{1}{4}\right) < \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$



$\left(-\frac{1}{2}, 1\right)$

(+) OR (−)

$x - \frac{1}{4} < \frac{3}{4}$

$\boxed{x < 1}$

$-\left(x - \frac{1}{4}\right) < \frac{3}{4}$

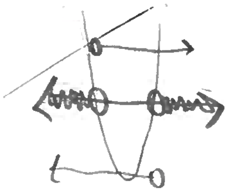
$-x + \frac{1}{4} < 3/4$

$-x < \frac{2}{4} = \frac{1}{2}$

mult by (−1)

$\boxed{x > -\frac{1}{2}}$

Ex: $x^2+x-10>0$ $\qquad 10=\frac{40}{4}$



Soln: compl. square, $b=1$

$$\left[\left(x+\tfrac{1}{2}\right)^2-\left(\tfrac{1}{2}\right)^2\right]-10>0$$

$$\left(x+\tfrac{1}{2}\right)^2>10+\tfrac{1}{4}=\tfrac{41}{4}$$

$$\Big\downarrow \text{sqrt}$$

$$\pm\left(x+\tfrac{1}{2}\right)>\frac{\sqrt{41}}{2}$$

$\oplus$:

$$x+\tfrac{1}{2}>\frac{\sqrt{41}}{2}$$

$$x>\frac{\sqrt{41}}{2}-\frac{1}{2}$$

$$\approx 2.70$$

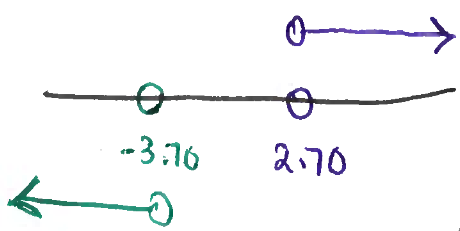
$\ominus$:

$$-\left(x+\tfrac{1}{2}\right)>\frac{\sqrt{41}}{2}$$

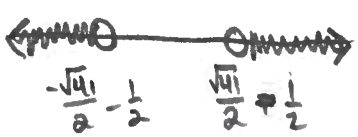
$$-x-\tfrac{1}{2}>\frac{\sqrt{41}}{2}$$

$$-x>\frac{\sqrt{41}}{2}+\frac{1}{2}$$

$$x<-\frac{\sqrt{41}}{2}-\frac{1}{2}\approx -3.70$$



$$\left(-\infty,-\frac{\sqrt{41}}{2}-\frac{1}{2}\right)\cup\left(\frac{\sqrt{41}}{2}-\frac{1}{2},\infty\right)$$



FACT: if r_1 and r_2 are solutions to $ax^2+bx+c=0$, then

$$ax^2+bx+c=(x-r_1)(x-r_2)\ \cancel{>}0$$

$$<0$$

>0: $\begin{pmatrix} x-r_1<0 \\ \text{and} \\ x-r_2<0\end{pmatrix}$ OR $\begin{pmatrix} x-r_1>0 \\ \text{and} \\ x-r_2>0\end{pmatrix}$

<0: $\begin{pmatrix} x-r_1>0 \\ \text{and} \\ x-r_2<0\end{pmatrix}$ OR $\begin{pmatrix} x-r_1<0 \\ \text{and} \\ x-r_2>0\end{pmatrix}$

This seems to be what we observe:

$x^2+bx+c>0$ $x^2+bx+c<0$

(complete square)

(isolate $(x+\frac{b}{2})^2$ term)

(take $\sqrt{\ }$)

keeping $\pm$ on the x-stuff

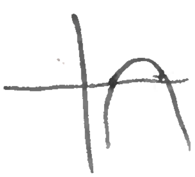
Solve each soln individually

soln of ineq. is union of both

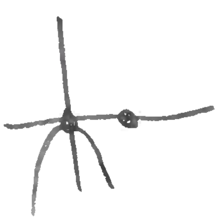
soln of inequality is intersection of both

Ex: $-x^2+2x-1>0$

$\downarrow$ mult by -1

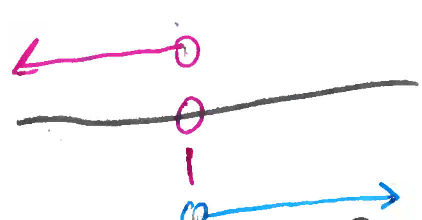


$(x^2-2x)+1<0$



$(x-1)^2-1+1<0$

$(x-1)^2<0$



$\pm(x-1)<0$

+ / −

$x-1<0$

$x<1$

$-x+1<0$

$-x<-1$

$x>1$

NO common $\cap$

$\downarrow$

NO SOLN

$(-1)(-x^2) = x^2$

Ex: $-x^2 + 3x + 100 > 0$

$\downarrow$ mult by -1

$(x^2 - 3x) - 100 < 0$

end up taking intersection

$100 + \frac{9}{4}$
$\parallel$
$\frac{400}{4} + \frac{9}{4}$

$x^2 + bx$

$b = -3$

$\frac{b}{2} = -3/2$

$x + \frac{b}{2}$

$\left(\frac{b}{2}\right)^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

$\downarrow$ compl. square

$\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} - 100 < 0$

$\downarrow$

$\left(x - \frac{3}{2}\right)^2 < \frac{409}{4}$

$\downarrow$

$\pm\left(x - \frac{3}{2}\right) < \frac{\sqrt{409}}{2}$

$+$

$x - \frac{3}{2} < \frac{\sqrt{409}}{2}$

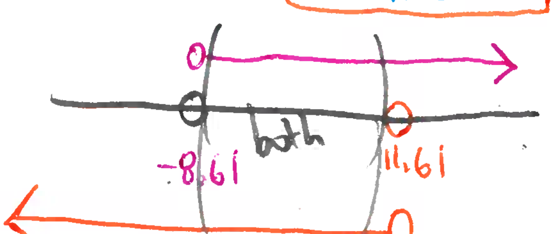
$\boxed{x < \frac{\sqrt{409}}{2} + \frac{3}{2} \approx 11.61}$

$-\left(x - \frac{3}{2}\right) < \frac{\sqrt{409}}{2}$

$-x + \frac{3}{2} < \frac{\sqrt{409}}{2}$

$-x < \frac{\sqrt{409}}{2} - \frac{3}{2}$

$\boxed{x > -\frac{\sqrt{409}}{2} + \frac{3}{2} \approx -8.61}$



$\rightarrow \left(-\frac{\sqrt{409}}{2} + \frac{3}{2}, \frac{\sqrt{409}}{2} + \frac{3}{2}\right)$