$b=-5$

Ex: $3x^2-5x+10=0$

$a=3$ $\qquad$ $c=10$

$-120+25=$

By QF,

$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(10)}}{2(3)}$$

$$=\frac{5\pm\sqrt{25-120}}{6}=\frac{5}{6}\pm\frac{\sqrt{-95}}{6}$$

$$=\frac{5}{6}\pm\frac{\sqrt{95}}{6}i$$

Quadratic inequalities

Recall linear inequalities:

$$ax+b<c$$

$$ax<c-b$$

$a>0$: $x<\frac{c-b}{a}$ $\qquad$ $a<0$: $x>\frac{c-b}{a}$

Rule if $a<0$

$b<c$

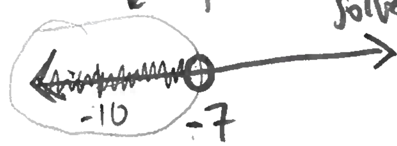
$\downarrow$

$ab>ac$

Ex: $-x-2>5$

$-x>5+2=7$

this set of numbers — $(-\infty,-7)$ — solves "$-x>7$"



$-(-7)>7$ false

$7>7$

$x=5$: "$-5>7$?" No - false

not in soln

$x=-10$

is in soln

$-(-10)>7$

$10>7$ YES TRUE

Soln to "$-x > 7$" is $(-\infty, -7)$

mult by (-1) $\longrightarrow$ $x < -7$ ✓

~~mult by (-1) (not flip > to <)~~

$\cancel{x > -7}$ ← WRONG

not same set!!

-7

Ex: $-1 < 1$

mult by (-1) $\longrightarrow$ $1 > -1$ true

~~mult by (-1) (not flip)~~

$\cancel{1 < -1}$ false

Ex: Solve $x^2 + 5x + 6 > 0$

"What x-values make the left side > 0?"

Left side factors:

$$(x+3)(x+2) > 0$$

When can product of these two factors be ⊕?

OR

① Could be that both $(x+3) > 0$ and $(x+2) > 0$ $\longrightarrow$ $x > -3$ and $x > -2$

$x > -3$

-3 -2

$x > -2$

② Could be that both $x + 3 < 0$ and $x + 2 < 0$ $\longrightarrow$ $x < -3$ and $x < -2$

-3 -2

Conclusion:

-3 -2

soln is $(-\infty, -3) \cup (-2, \infty)$

Ex: Same problem but w/o factoring:

$$(x^2+5x)+6>0$$

$\downarrow$ complete square

$$\boxed{x^2+5x=\left(x+\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2}$$

$b=5$

$$\left(x+\frac{5}{2}\right)^2=\frac{1}{4}$$

$$\pm\left(x+\frac{5}{2}\right)=\pm\sqrt{\frac{1}{4}}$$

$$-\frac{25}{4}+6=-\frac{25}{4}+\frac{24}{4}$$

$$=-\frac{1}{4}$$

So, $x^2+5x+6>0$

$$\left(x+\frac{5}{2}\right)^2-\frac{25}{4}+6>0$$

$$\left(x+\frac{5}{2}\right)^2-\frac{1}{4}>0$$

take sqrt $\quad \left(x+\frac{5}{2}\right)^2>\frac{1}{4}$

$$\pm\left(x+\frac{5}{2}\right)>\sqrt{\frac{1}{4}}=\frac{1}{2}$$

$$-x-\frac{5}{2}>\frac{1}{2} \qquad\qquad x+\frac{5}{2}>\frac{1}{2}$$

$$-x>\frac{1}{2}+\frac{5}{2} \qquad\qquad x>\frac{1}{2}-\frac{5}{2}=-\frac{4}{2}=-2$$

$$-x>\frac{6}{2}=3$$

$$-x>3$$

$\downarrow$ m. by -1

$$x<-3$$

1-60

Ex: $x^2 - x + 15 > 0$

$x^2 - x + \frac{1}{4} - \frac{1}{4}$

Aside

QF

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(15)}}{2}$

$= \frac{1}{2} \pm \frac{1}{2}\sqrt{-59}$

$= \frac{1}{2} \pm \frac{\sqrt{59}}{2} i$

Complete square: $x^2 - x = \left(x - \frac{1}{2}\right)^2 - \left(\frac{-1}{2}\right)^2$

$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) = \frac{1}{4}$

So,

$x^2 - x + 15 > 0$

Same as

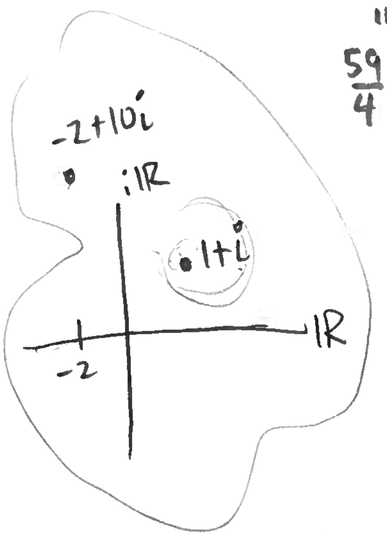
$$\left(x - \frac{1}{2}\right)^2 \underbrace{- \left(\frac{1}{4}\right) + 15}_{= -\frac{1}{4} + \frac{60}{4} = \frac{59}{4}} > 0 \Rightarrow \left(x - \frac{1}{2}\right)^2 + \frac{59}{4} > 0$$

$$\Rightarrow \boxed{\left(x - \frac{1}{2}\right)^2 > -\frac{59}{4}}$$

FACT IS: any real number squared is ≥ 0

left side can never be negative unless possibly x is a complex #

we don't consider complex #'s when doing inequalities



Why?

It's difficult (impossible) for the complex #'s to be ordered AND maintain their properties

Conclusion: solution is all $\mathbb{R} = (-\infty, \infty)$