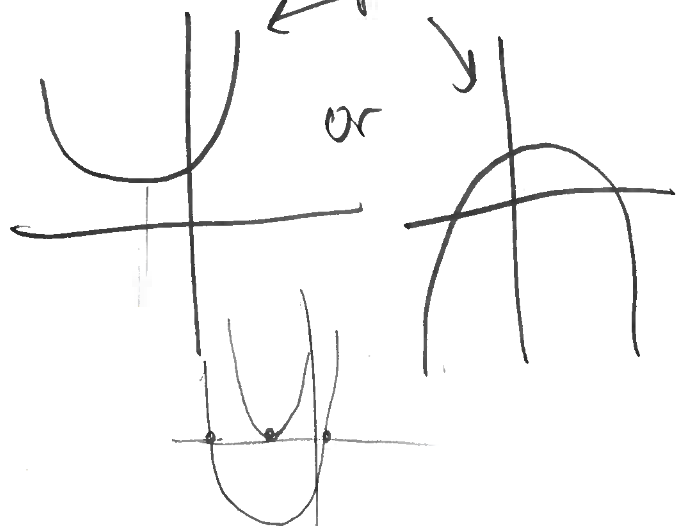
Quadradic functions

earlier: $f(x) = ax + b$

x^2 term makes it "quadratic"

now: $f(x) = ax^2 + bx + c$ ~ generally look like parabolas



Quadratic equation

$$ax^2 + bx + c = d$$

Question: How to solve $ax^2 + bx + c = d$?

ALWAYS: "quadratic formula" will do it

$ax + b = d$

$ax = d - b$

$x = \frac{d-b}{a}$

Ex: $x^2 - 1 = 0$

$x^2 = 1$

$x = \pm\sqrt{1} = 1, -1$

(Recall: $\sqrt{x^2} = x$
square roots are inverse of x^2)

"$\sqrt{\ }$" ← always means $\oplus$ sq. root

Ex: $x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1}=\pm i$

$i^2+1=(\sqrt{-1})^2+1$
$=-1+1=0$ ✓

$x^2+3x=-2$ can't isolate x

~~$x(x+3)=-2$~~

~~$x=-2$ OR $x+3=-2$~~

Ex: $x^2+3x+2=0$

can't solve for x "like before"

idea: Reduce this to a $\underbrace{\text{problem I can solve}}_{\text{often linear}}$

"zero product property" ⇝ $a\cdot b=0$ ← 0 is special!!

$a=0$ OR $b=0$

Big plan: express

$0=x^2+3x+2=\underbrace{(x+p)(x+q)}$ linear (multiplied so I get an "x^2" out of it)

"ansatz" ~ "desired form"

goal: find p and q

dist law

$0=x^2+3x+2=(x+p)x+(x+p)q$
$=x^2+(px+qx)+pq$
$=x^2+(p+q)x+pq$

match!

must force to match

force to match

$\Rightarrow \boxed{\begin{cases}3=p+q\\2=pq\end{cases}}$

THINK

p	q	pq=2	p+q
1	2	2 ✓	3
2	1	2 ✓	3

From chart: $p=1$, $q=2$ should work.

Check: Ansatz $\to$ $(x+1)(x+2) = (x+1)x + (x+1)2$

$= x^2 + x + 2x + 2$

$= x^2+3x+2$ ✓

works!!

So now solving $\underbrace{x^2+3x+2}_{\text{parabola}} = \underbrace{0}_{\text{horizontal line}}$

is same as solving

$(x+1)(x+2) = 0$

zero prod prop

$x+1=0$ OR $x+2=0$

$\boxed{x=-1}$ $\quad$ $\boxed{x=-2}$

Ex: Solve $x^2+4x-5=0$

negative

$5 = 1 \cdot 5$

$(x - 1)(x+5) = 0$

5 1 $\qquad$ 1 5

$x-1=0$ OR $x+5=0$

$\boxed{x=1}$ $\quad$ $\boxed{x=-5}$

Ex: $x^2 + x + 1 = 0$

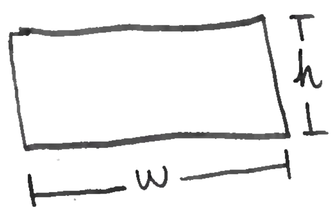
$1 = 1 \cdot 1$ $\quad (x + \quad)(x + \quad) = 0$

fails!

We will come back to this

UNFORTUNATE FACT: majority of quadratic polynomials do not factor nicely!!

Recall:

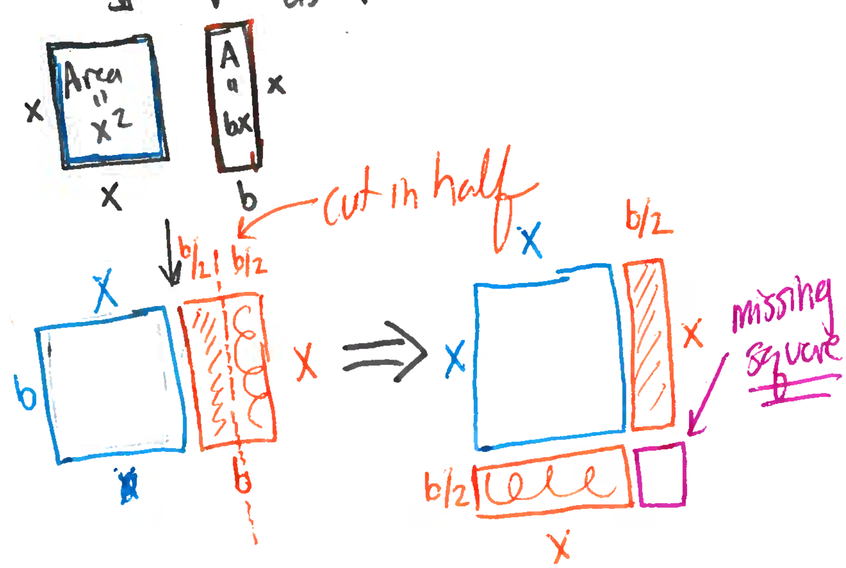


$\text{Area} = w \cdot h$

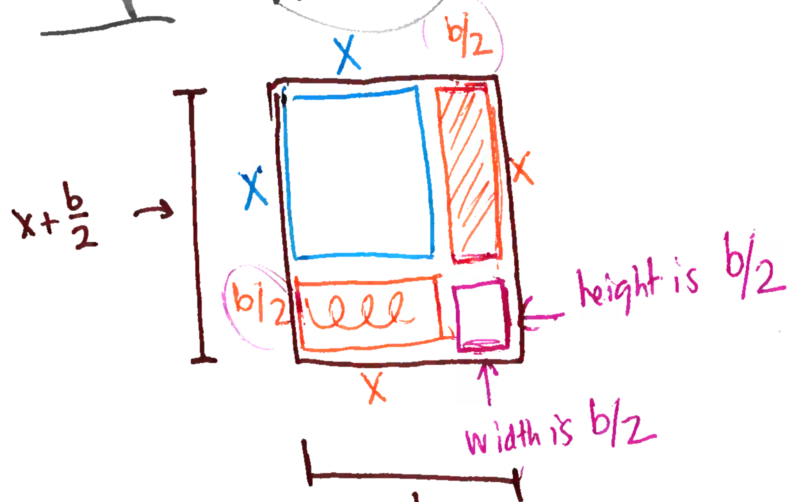
Goal: somehow "factor" $x^2 + bx$

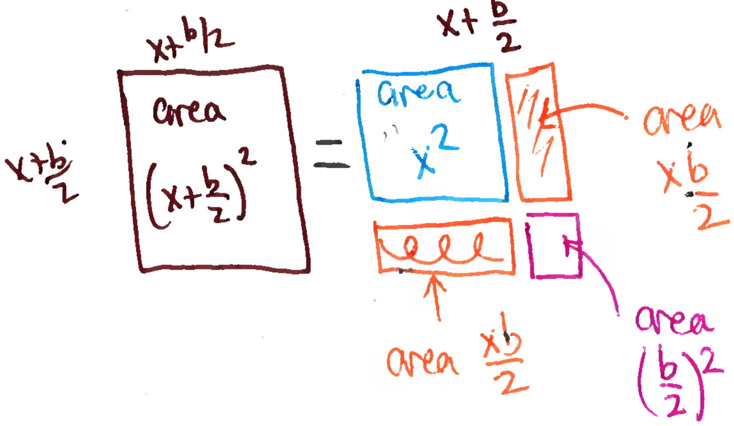
$x \cdot x$ $\quad b \cdot x$

interpret as areas



Recap: x^2+bx





$$\left(x+\frac{b}{2}\right)^2 = x^2 + \underbrace{\frac{xb}{2}+\frac{xb}{2}}_{=xb} + \left(\frac{b}{2}\right)^2$$

So,

$$\boxed{\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2 = x^2+bx}$$

Back to ex

$$(x^2+x)+1=0$$

x^2+bx
w/ $b=1$

So by trick we did:

$$(x^2+x) = \left(x+\tfrac{1}{2}\right)^2 - \left(\tfrac{1}{2}\right)^2$$

So,

$$(x^2+x)+1=0$$

$$\left(x+\tfrac{1}{2}\right)^2 \underbrace{-\tfrac{1}{4}+1}_{=3/4}=0$$

$$\left(x+\tfrac{1}{2}\right)^2 = -\frac{3}{4} \longrightarrow x+\frac{1}{2} = \pm\sqrt{\frac{-3}{4}} = \pm\sqrt{\frac{3}{4}}\, i$$

$$\longrightarrow x = -\frac{1}{2} \pm \sqrt{\frac{3}{4}}\, i$$