

Ex: Given $f(x) = 3x + 5$,

①

find $f^{-1}(x)$.

↓ solve for x

Soln:

$$y = 3x + 5$$

$$x = 3y + 5$$

$$x - 5 = 3y$$

$$\frac{x-5}{3} = y$$

$$y - 5 = 3x$$

$$\frac{y-5}{3} = x$$

$$f^{-1}(x) = \frac{x-5}{3}$$

Is this actually f^{-1} ?

check if $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

Calculate

$$f(f^{-1}(x)) = f\left(\frac{x-5}{3}\right)$$

$$= \left(\frac{x-5}{3}\right) \cdot 3 + 5$$

$$= x - 5 + 5 = x + 0 = x \checkmark$$

Calculate

$$\hookrightarrow f^{-1}(f(x)) = f^{-1}(3x+5)$$

$$= \frac{(3x+5) - 5}{3} = \frac{3x}{3} = 1x = x \checkmark$$

Ex: $g(x) = \frac{x-7}{5}$ find $g^{-1}(x)$

(2)

$$y = \frac{x-7}{5}$$

$$5y = x-7$$

$$x = 5y + 7$$

$$\Rightarrow g^{-1}(x) = 5x + 7$$

Check

$$g(g^{-1}(x)) = g(5x+7) = \frac{(5x+7)-7}{5} = \frac{5x}{5} = x \checkmark$$

$$g^{-1}(g(x)) = g^{-1}\left(\frac{x-7}{5}\right) = 5\left(\frac{x-7}{5}\right) + 7 = x-7+7 = x \checkmark$$

Ex: Find inverse for

$h: D \rightarrow \mathbb{R}$

(3)

$h(x) = 3 + \sqrt{5+2x}$

can't have negative under the $\sqrt{\quad}$

$\text{dom}(h) = [-\frac{5}{2}, \infty)$
 $\text{range}(h) = [3, \infty)$

require
 $5+2x \geq 0$
 $2x \geq -5$
 $x \geq -\frac{5}{2}$

$y = 3 + \sqrt{5+2x}$

solve for x

$y-3 = \sqrt{5+2x}$

square both sides

$(y-3)^2 = (\sqrt{5+2x})^2$

$(y-3)^2 = 5+2x$

$(y-3)^2 - 5 = 2x$

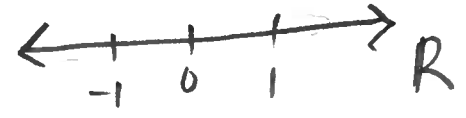
$x = \frac{(y-3)^2 - 5}{2}$

$h^{-1}(x) = \frac{(x-3)^2 - 5}{2}$

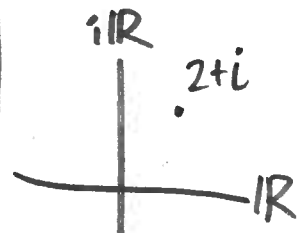
$\text{dom}(h^{-1}) = \text{range}(h) = [3, \infty)$

$\text{range}(h^{-1}) = \text{dom}(h) = [-\frac{5}{2}, \infty)$

$\sqrt{-2} = \sqrt{2}i$
 $i = \sqrt{-1}$

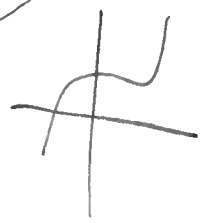


i is not a "real number"



$f: \mathbb{C} \rightarrow \mathbb{C}$
 2Dim 2Dim

$f: \mathbb{R} \rightarrow \mathbb{R}$
 1D 1D



EX:

$C(F) = C = \frac{5}{9}(F - 32)$
 ↑ name ↑ var ↑ Celsius temp. ↑ Fahrenheit
 $F \geq -459.67$
 ↑
 $\text{dom}(C(F)) = [-459.67, \infty)$
 $\text{range}(C(F)) = [-273.15, \infty)$
Inverse

EX: 72°F

↓
 $C = \frac{5}{9}(72 - 32)$
 $= \frac{5}{9}(40)$
 $= \frac{200}{9} = 22.2^\circ\text{C}$
 $72^\circ\text{F} = 22.22^\circ\text{C}$



32°F freezing } water
 212°F boiling }

0°C freeze } water
 100°C boi }

$$y = \frac{5}{9}(F - 32)$$

$$\frac{9}{5}y = F - 32$$

$$F = 32 + \frac{9}{5}y$$

$$\Rightarrow C^{-1}(F) = 32 + \frac{9}{5}F$$

$$\text{dom}(C^{-1}) = \text{range}(C) = [-273.15, \infty)$$

$$\text{range}(C^{-1}) = \text{dom}(C) = [-459.67, \infty)$$