New minor → Data Science

offered every sem → 2 low level: MATH 1550 (baby stats) (satisfies Tech column in core curriculum); COMP 1110 / Python — 2 years

first offered Fall/Spr next yr → 2 medium level: 2 2000-level math/cs courses

2 upper level: capstone (project); Data Mining

Compositions of functions

Plugging a function into another

↓

We've seen it!

For ex: $f(x) \longrightarrow f(x+2)$

$f(x+2)$: composition of $f(x)$ and $g(x) = x+2$

Notation: $(f \circ g)(x) = f(g(x))$

"of"

"f of g"

how to compute

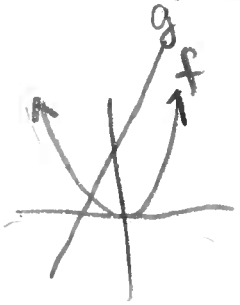
f of f of ... of f

800 times

"dynamical system"

leads to fractals/etc

used to solve practical problem



Ex: If $f(x) = x^2$ and $g(x) = x+7$, then compute $(f \circ g)(x)$ and $(g \circ f)(x)$.
State domain of f, g, $f \circ g$, and $g \circ f$.

Soln: $\text{dom}(f) = \mathbb{R}$

$\text{dom}(g) = \mathbb{R}$

$\text{dom}(f \circ g) = \mathbb{R}$

$(f \circ g)(x) = f(g(x)) = f(x+7) = (x+7)^2$

(def)

NOT SAME

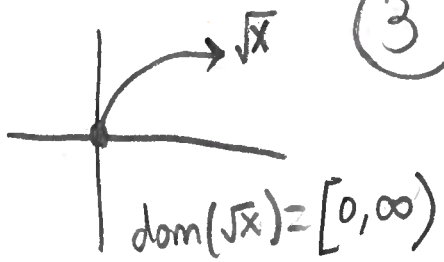
$(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 + 7$

$\text{dom}(g \circ f) = \mathbb{R}$

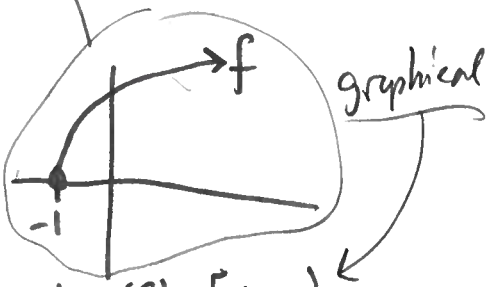
Ex: Same instr but $f(x) = \sqrt{x+1}$

$g(x) = x-5$

$\text{dom}(g) = \mathbb{R} = (-\infty, \infty)$

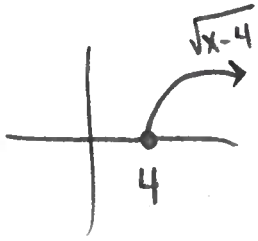


need $x+1 \geq 0$, $x \geq -1$ (symbolic)



$\text{dom}(f) = [-1, \infty)$

$(f \circ g)(x) \underset{\text{def}}{=} f(g(x))$



$= f(x-5)$

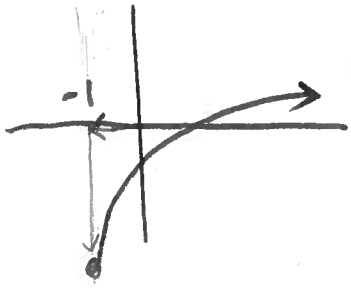
$= \sqrt{(x-5)+1}$

$= \sqrt{x-4}$

$\text{dom}(f \circ g) = [4, \infty)$

$(g \circ f)(x) \underset{\text{def}}{=} g(f(x)) = g(\sqrt{x+1}) = (\sqrt{x+1}) - 5$

$\text{dom}(g \circ f) = [-1, \infty)$



Ex: $f(x)=\frac{1}{x-1}$ $g(x)=\sqrt{2x+3}=\sqrt{2\left(x+\frac{3}{2}\right)}$

h. comp ↑ h. shift left ↑

domain must not allow us to divide by zero

$\Downarrow$

$x-1\neq 0$

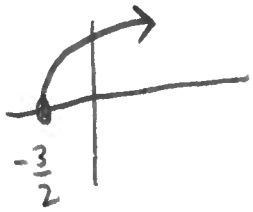
$x\neq 1$

$\text{dom}(f)=\mathbb{R}\setminus\{1\}$

$=(-\infty,1)\cup(1,\infty)$

must not let insides of $\sqrt{\ }$ be negative

symbolic / graphical

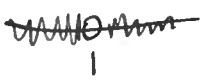


Require $2x+3\geq 0$

$2x\geq -3$

$x\geq -\frac{3}{2}$

$\left[-\frac{3}{2},\infty\right)$



$(f\circ g)(x)=f(g(x))=f\left(\sqrt{2x+3}\right)=\frac{1}{\sqrt{2x+3}-1}$

↑ def

we know $x\geq -3/2$

also

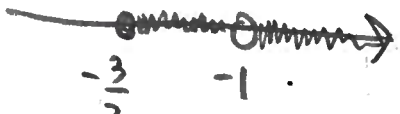
$\sqrt{2x+3}-1\neq 0$

$\sqrt{2x+3}\neq 1$

$2x+3\neq 1$

$2x\neq -2$

$x\neq -1$



$\text{dom}(f\circ g)=\left[-\frac{3}{2},-1\right)\cup(-1,\infty)$

FINISH !!