

Function transformations

①

Vertical shift

if you know graph of $f(x)$,



then, the graph of $f(x)+2$



In general, graph of

$$f(x) \pm C$$

is the same graph of $f(x)$, shifted vertically by C units (~~up if~~ $C > 0$) ~~down if~~ $C < 0$)

Action: add C to y -values in graph

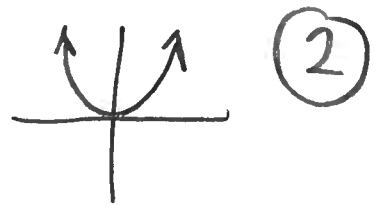
Horizontal shift

graph of $f(x \pm c)$ ($c > 0$)

~~Might think: $f(x+c)$ moves graph right by c~~
false!

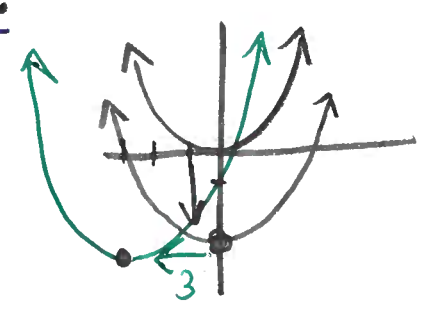
Actuality: $f(x+c)$ moves graph left } "horizontal is twisted"
 $f(x-c)$ moves graph right }

Ex: Suppose $f(x) = x^2$ and graph of $g(x)$ is graph of $f(x)$ shifted vertically down by 2 + horizontally left by 3.



Find formula for $g(x)$

$f(x) - 2$ $f(x+3)$



So it must be that

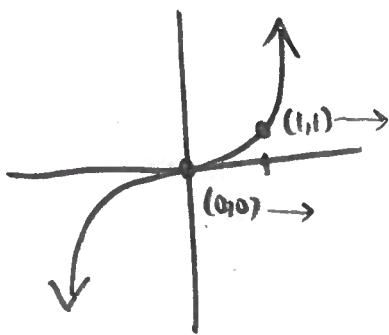
$$g(x) = f(x+3) - 2$$

$$= (x+3)^2 - 2$$

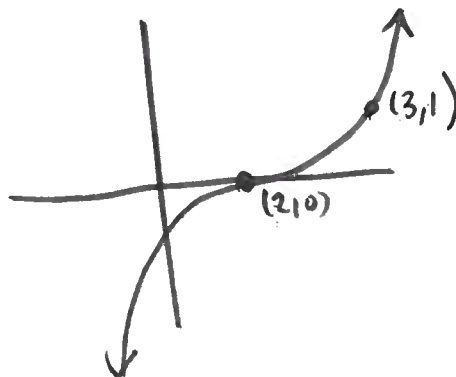
$$= x^2 + 6x + 9 - 2$$

$$= x^2 + 6x + 7$$

Ex: Here is $f(x)$:



Here is $g(x)$



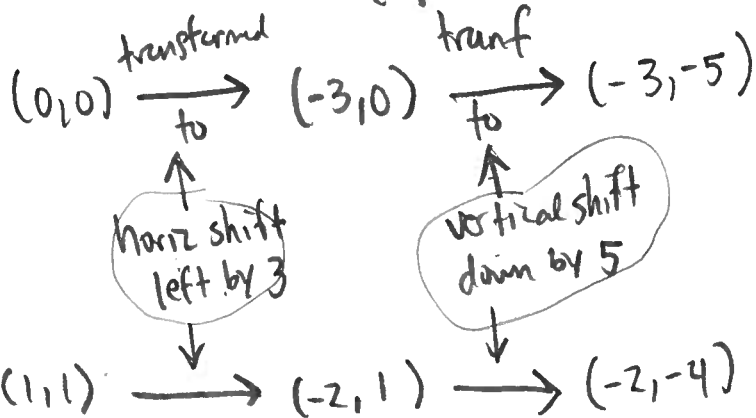
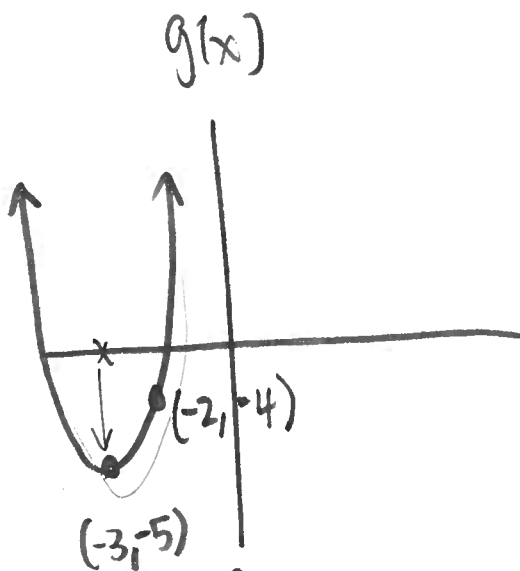
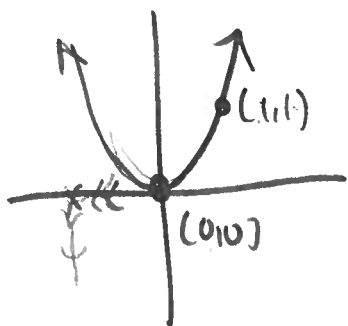
What is $g(x)$?

$$g(x) = f(x-2)$$

3

EX:

$$f(x) = x^2$$

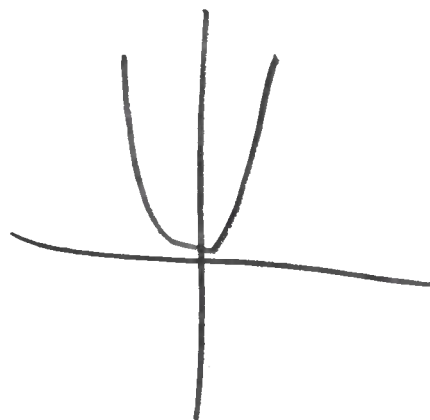
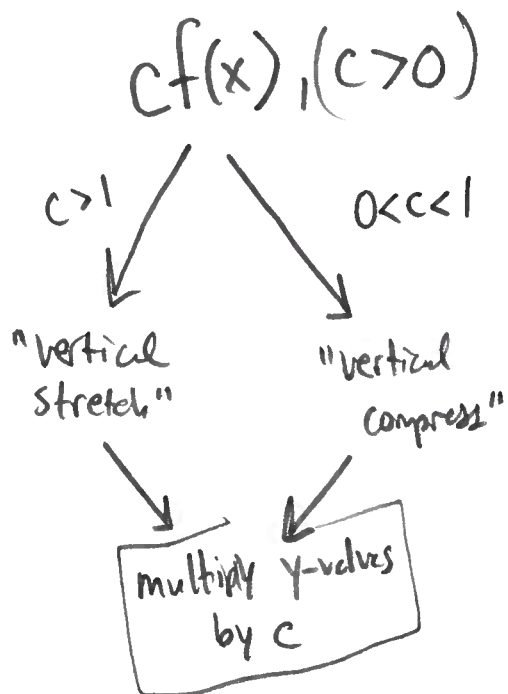


So,

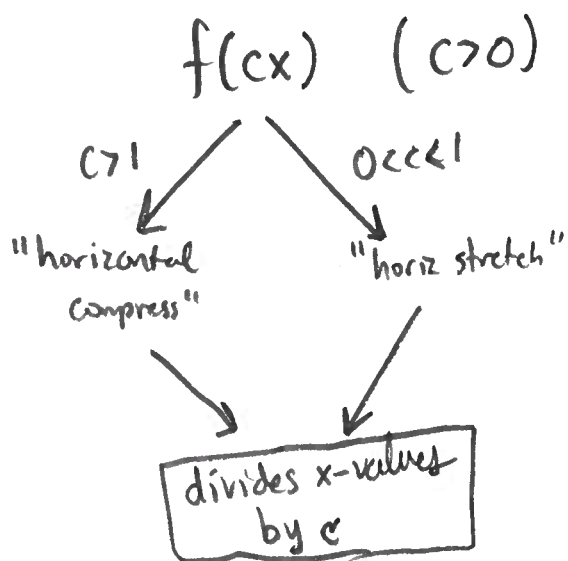
$$\begin{aligned} g(x) &= f(x+3) - 5 = (x+3)^2 - 5 \\ &= x^2 + 6x + 9 - 5 \\ &= x^2 + 6x + 4 \end{aligned}$$

~~Horizontal~~ stretch/compress
Vertical

4



Horizontal stretch/compress



Ex: Describe what effect

$$y = 5f(x-2)$$

has on graph of f_0

~~horizontal~~
vertical stretch
by factor of 5

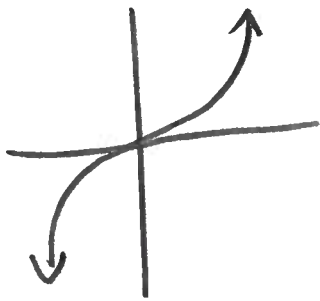
horizontal shift
~~left~~ by 2
right

Reflections

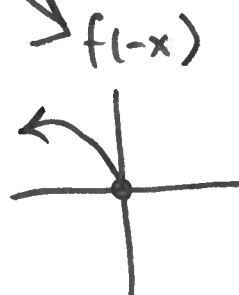
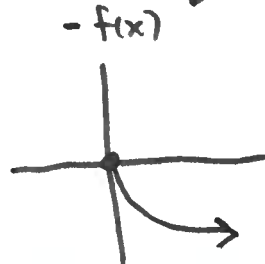
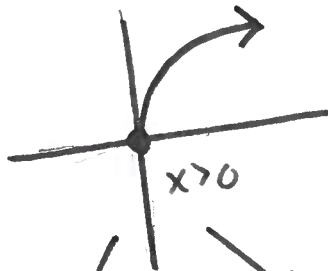
$-f(x)$ ~ vertical reflection ~ mirror image of graph across x-axis

$f(-x)$ ~ horizontal reflection ~ mirror image of graph across y-axis

$$f(x) = x^3$$

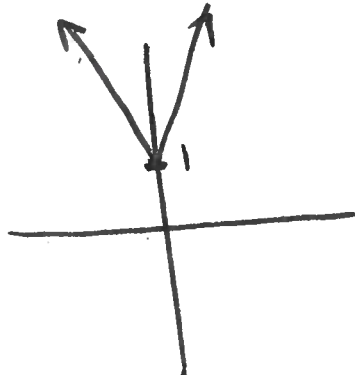


$$f(x) = \sqrt{x}$$

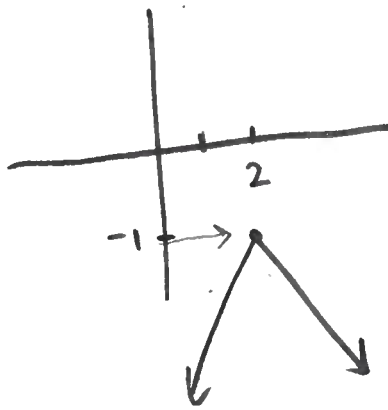


Ex: Here is $f(x) = |x| + 1$

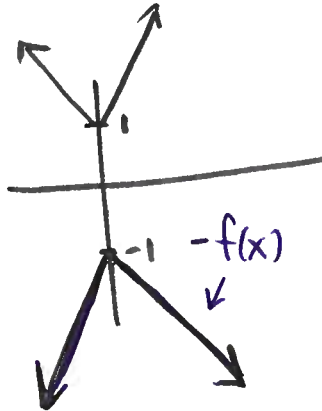
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Find formula for



vertical reflection



horiz shift right

