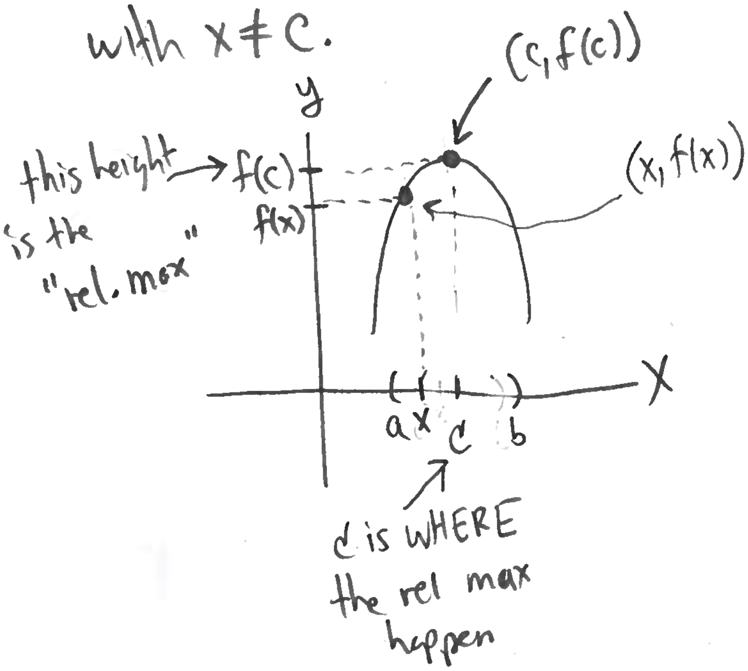
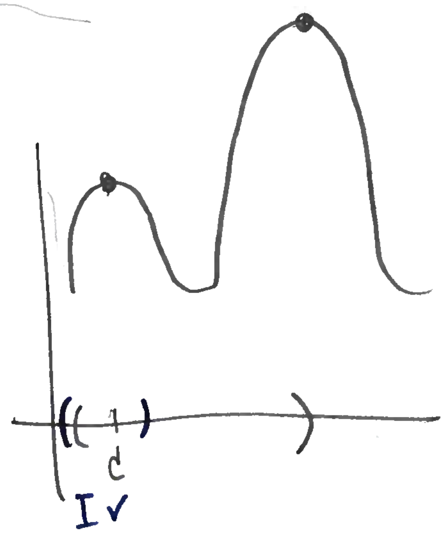
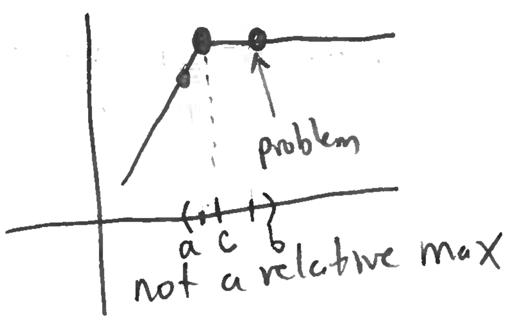
<u>Definition</u>: We say $f(c)$ is a relative maximum if $\boxed{\text{there is}}$ an interval $I=(a,b)$ containing c such that $f(c) > f(x)$ for all x in I with $x \neq c$.



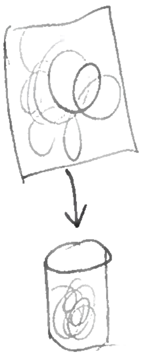


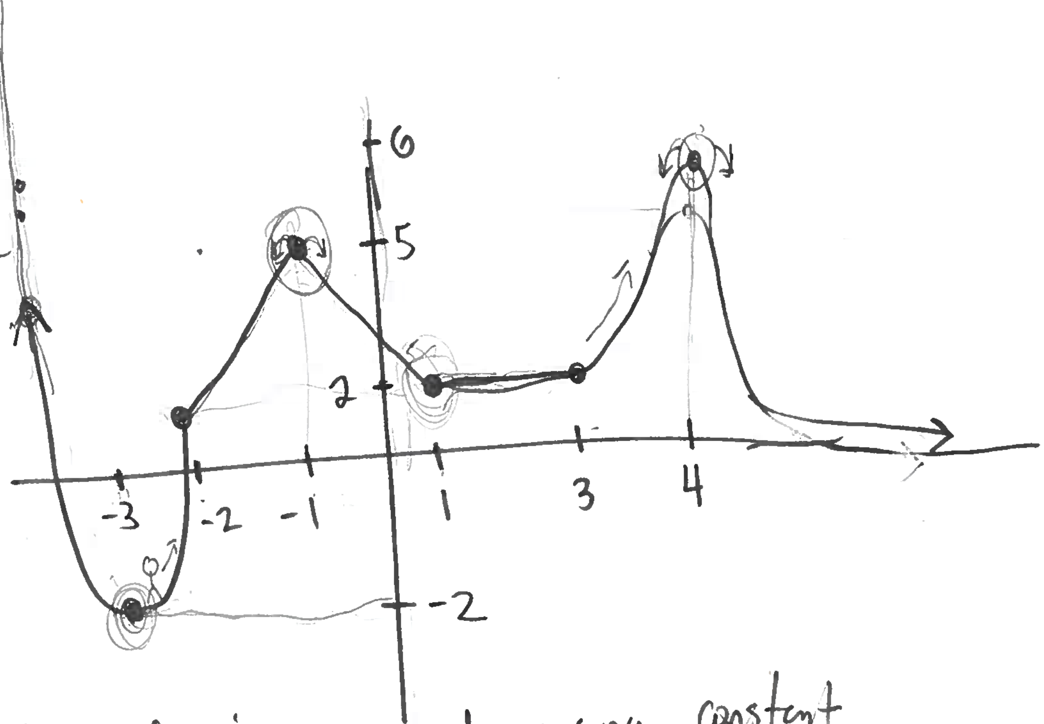


Rel min is same, except you use "$f(c) < f(x)$"

②

Ex:





Identify increasing, decreasing, constant
+ any relative maximum or relative min.
+ where they occur.
+ Domain + range.

Soln: domain: $(-\infty, \infty)$

range: $[-2, \infty)$

separate intervals w/ commas

where increasing: $(-3,-2), (-2,-1), (3,4)$

where decreasing: $(-\infty,-3), (-1,1), (4,\infty)$

where constant: $(1,3)$

rel max: 5, 6

rel min: -2,

where rel max: 1, 4

where rel min: -3

even numbers: 2, 4, 6, 8, 0, -6

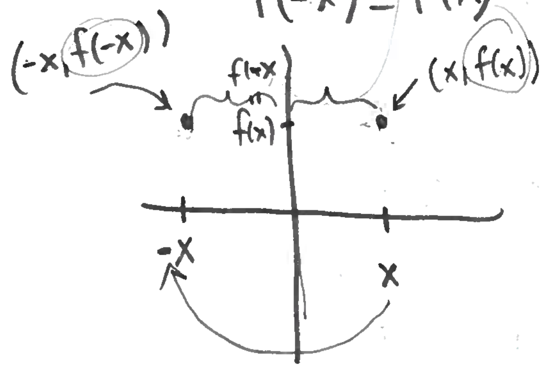
odd numbers: 1, -1, 3, 5, 7, ...

Symmetry

even function

A function where for all x in domain,

$$f(-x) = f(x)$$

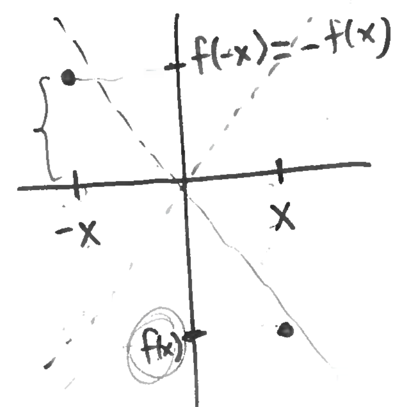


even function is "symmetric with respect to y-axis"

odd function

A function where for all x in domain,

$$f(-x) = -f(x)$$

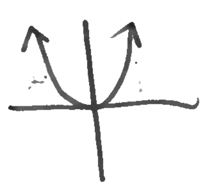


"symmetric about line y=x"

"symmetric about origin"

Origin of this terminology

Polynomials —



$f(x) = x^{2}$ ← even power

$f(2) = 2^2 = 4$

$f(-2) = (-2)^2 = (-2)(-2) = 4$

$\underbrace{f(-x)}_{\text{start}} = (-x)^2 = (-x)(-x) = x^2 = \underbrace{f(x)}_{\text{end}}$

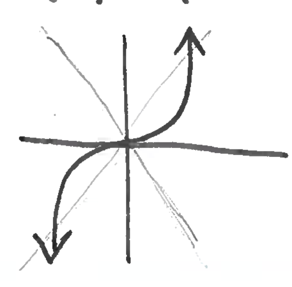
Generally: $f(x) = x^m$ where m is an even # is an even fnct

$f(x) = x^3$

$f(2) = 2^3 = 8$

$f(-2) = (-2)^3 = (-2)(-2)(-2)$
$= -8$

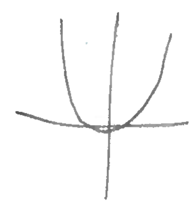
$f(-2) = -f(2)$



Ex: Even, odd, or neither?

a) $f(x) = x^4 + x^2 + x^{10}$

b) $g(x) = \dfrac{x^3 - x}{x^5}$



c) $h(x) = 2x^2 + x^3$

Soln: (a) $f(-x) = (-x)^4 + (-x)^2 + (-x)^{10}$

$= x^4 + x^2 + x^{10}$ ← because even powers eat up minus signs!

$= f(x)$

$\Rightarrow f$ is even

(b) $g(-x) = \dfrac{(-x)^3 - (-x)}{(-x)^5}$

$\dfrac{a}{-b} = -\dfrac{a}{b} = \dfrac{-a}{b}$

$= \dfrac{-x^3 - (-x)}{-x^5}$ ← because odd powers move minus sign outside

$= \dfrac{-(x^3 - x)}{-x^5}$

$= \dfrac{x^3 - x}{x^5} = g(x)$

$\Rightarrow$ even

(c) $h(-x) = 2(-x)^2 + (-x)^3 = 2x^2 - x^3$ $\Rightarrow$ neither odd nor even

↑ neither $h(x)$ nor $-h(x)$

Application to trig + calc 2

$n! = n(n-1)\cdots(2)(1)$

$4! = 4\cdot3\cdot2\cdot1 = 24$

odd fnct

sine $\longrightarrow \sin(x) =$ "infinite polynomial" $= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

cosine $\longrightarrow \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$

even fnct

Next $\rightarrow$ function transformation

$\Downarrow$

graphing!