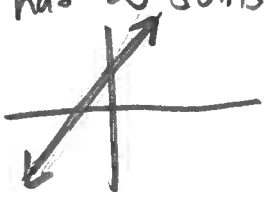
Systems of two linear equations

$2x+1=5$ — one eqt, one var → one soln

→ **earlier:** $y = 2x+3$ ← has ∞ solns — $(0,3)$

$3 = 2(0)+3$ TRUE



one eqt two var → ∞ solns

Now:

$$\begin{cases} ax+by=c \\ dx+eg=f \end{cases}$$

$(x, y$ variables$)$

two eqt two vars → expect: one soln (or no soln)

a solution of this system is a choice of variables that make BOTH eqts true

Ex:

$$\begin{cases} x-y=2 \quad (i) \\ x+y=5 \quad (ii) \end{cases}$$

add vertically

$2x+0=7$

$2x=7$

$x=\frac{7}{2}$

$2=\frac{4}{2}$

$5=\frac{10}{2}$

(i) $\frac{7}{2}-y=2$

$\frac{7}{2}-2=y$

$\frac{3}{2}=y$

(ii) $\frac{7}{2}+y=5$

$y=\frac{10}{2}-\frac{7}{2}=\frac{3}{2}$

Third way (plot)

substitution

Solve (i) for x:

$\boxed{x=2+y}$

Plug this into (ii):

$(2+y)+y=5$

"x"

$2+2y=5$

$\boxed{y=\frac{3}{2}}$

$x=2+\frac{3}{2}$

$=\frac{4}{2}+\frac{3}{2}$

$=\frac{7}{2}$

$\Rightarrow$ soln is $(x,y)=(\frac{7}{2},\frac{3}{2})$

Ex: $\begin{cases} 3x-2y=1 & \text{(i)} \\ 2x+3y=2 & \text{(ii)} \end{cases}$

$2\left(\frac{3}{2}\right)=3$

(iii): $\frac{3}{2}$(i) $\quad \frac{9}{2}x-3y=\frac{3}{2}$

subst.

$\frac{3x-1}{2}=\frac{3x}{2}-\frac{1}{2}$

add vert

$\frac{13}{2}x+0=\frac{7}{2}$

$x=\frac{14}{26}=\frac{7}{13}$

↓(ii)

$\frac{14}{13}+3y=2$

$\frac{26}{13}$

$3y=\frac{26}{13}-\frac{14}{13}=\frac{12}{13}$

$y=\frac{12}{3\cdot 13}=\frac{4}{13}$

Solve (i) for y:

$3x-1=2y$

$\frac{3}{2}x-\frac{1}{2}=y$

↓ plug into (ii)

$2x+3\left(\frac{3}{2}x-\frac{1}{2}\right)=2$

$2x+\frac{9}{2}x-\frac{3}{2}=2$

$\frac{4}{2}x$

$\frac{13}{2}x-\frac{3}{2}=\frac{4}{2}$

$\frac{13}{2}x=\frac{7}{2}$

$13x=7$

$x=\frac{7}{13}$

$\frac{3}{2}\left(\frac{7}{13}\right)-\frac{1}{2}=y$

$\frac{21}{26}-\frac{13}{26}=y$

$\frac{8}{26}=y=\frac{4}{13}$

(x,y) =

soln: $\left(\frac{7}{13}, \frac{4}{13}\right)$

Ex: Two job offers: sales job

→ ① straight commission of 10% of all sales

② $300/wk plus 5% commission on all sales

How much do you have to sell to make the first job more profitable?

Soln: Recall ~ % "percent" — divide by 100

per — divide

cent — 100

$$\% = \frac{1}{100}$$

① income = 10% of sales

$$= \left(\frac{10}{100}\right)(A) = \frac{1}{10}A$$

$$\boxed{I = \frac{1}{10}A}$$

② income = 300 + 5% of sales

$$= 300 + \left(\frac{5}{100}\right)A$$

$$= 300 + \frac{1}{20}A$$

$$\boxed{I = 300 + \frac{1}{20}A}$$

Q: At what A does ① have larger I.

miles per hour

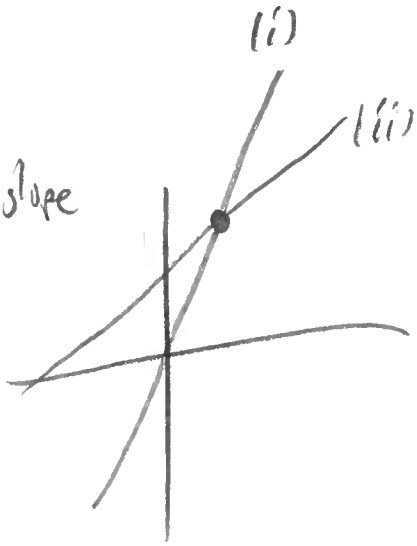
$\frac{30\text{ mi}}{2\text{ hr}} = 15\text{mph}$

1,000,000

(9)

$$\begin{cases} I = \frac{1}{10}\Delta & \text{(i)} \\ I = 300 + \frac{1}{20}\Delta & \text{(ii)} \end{cases}$$

bigger slope



↓ subst.

Plug (i) into (ii)

$\frac{1}{10} = \frac{2}{20}$ → $\frac{1}{10}\Delta = 300 + \frac{1}{20}\Delta$

→ $\frac{2}{20}\Delta - \frac{1}{20}\Delta = 300$

$\left(\frac{2}{20} - \frac{1}{20}\right)\Delta$

$\frac{1}{20}\Delta = 300$

$\Delta = 20(300) = 6000$

↑

$\left(\frac{20}{20}\right)\Delta = 300(20)$

Conclusion: any amount > \$6000 will make job ① more profitable to you

Ex: Biz ships 100 packages in a day.

They charge $2 for normal shipping ~ x (# of that type)

and $10 for one-day shipping. ~ y

They made ~$500 that day.

How much of each shipping type were sent?

$$\begin{cases} 2x + 10y = 500 & \text{(i)} \\ x + y = 100 & \text{(ii)} \end{cases}$$

(ii) → $x = 100 - y$

↓ (i)

$2(100 - y) + 10y = 500$

$200 - 2y + 10y = 500$

$200 + 8y = 500$

$8y = 300$

$y = \frac{300}{8}$

$x = 100 - \frac{300}{8}$

$= \frac{500}{8}$

$(x, y) = \left(\frac{500}{8}, \frac{300}{8}\right)$

$= (62.5, 37.5)$