HW8 MATH 3520 Fall 2019

#15) Given: $T: \mathbb{R}^{2\times 1} \to \mathcal{P}_2$

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = 1-2x \quad \text{and} \quad T\left(\begin{bmatrix}3\\-1\end{bmatrix}\right) = x+2x^2$$

Write $\begin{bmatrix}a\\b\end{bmatrix}$ in terms of the basis $\mathcal{B} = \left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}3\\-1\end{bmatrix}\right\}$:

$$c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}3\\-1\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$$

$$\left[\begin{array}{cc|c}1 & 3 & a\\1 & -1 & b\end{array}\right] \sim \left[\begin{array}{cc|c}1 & 0 & \frac{a}{4}+\frac{3}{4}b\\0 & 1 & \frac{a}{4}-\frac{b}{4}\end{array}\right]$$

The particular vector

$$\begin{bmatrix}-7\\9\end{bmatrix} = \left(\frac{-7}{4}+\frac{27}{4}\right)\begin{bmatrix}1\\1\end{bmatrix} + \left(\frac{-7}{4}-\frac{9}{4}\right)\begin{bmatrix}3\\-1\end{bmatrix}$$

$$= \frac{20}{4}\begin{bmatrix}1\\1\end{bmatrix} + \left(\frac{-16}{4}\right)\begin{bmatrix}3\\-1\end{bmatrix}$$

$$= 5\begin{bmatrix}1\\1\end{bmatrix} - 4\begin{bmatrix}3\\-1\end{bmatrix}$$

Therefore,

$$T\left(\begin{bmatrix}-7\\9\end{bmatrix}\right) = T\left(5\begin{bmatrix}1\\1\end{bmatrix} + (-4)\begin{bmatrix}3\\-1\end{bmatrix}\right)$$

$$\overset{\text{linearity}}{=} 5T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) - 4T\left(\begin{bmatrix}3\\-1\end{bmatrix}\right)$$

$$\overset{\text{given}}{=} 5(1-2x) - 4(x+2x^2)$$

$$= 5 - 10x - 4x - 8x^2$$

$$= -8x^2 - 14x + 5, \quad \text{and}$$

$$T\left(\begin{bmatrix}a\\b\end{bmatrix}\right) = T\left(\left(\frac{a+3b}{4}\right)\begin{bmatrix}1\\1\end{bmatrix} + \left(\frac{a-b}{4}\right)\begin{bmatrix}3\\-1\end{bmatrix}\right)$$

$$\overset{\text{linearity}}{=} \left(\frac{a+3b}{4}\right)T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) + \left(\frac{a-b}{4}\right)T\left(\begin{bmatrix}3\\-1\end{bmatrix}\right)$$

$$\overset{\text{given}}{=} \left(\frac{a+3b}{4}\right)(1-2x) + \left(\frac{a-b}{4}\right)(x+2x^2)$$

#27) $(S \circ T)(p(x)) = S(T(p(x))) = S(p'(x)) = p'(x+1)$

$(T \circ S)(p(x)) = T(S(p(x)) = T(p(x+1)) = \frac{d}{dx} p(x+1) = (p'(x+1)) \frac{d}{dx}(x+1)$

$= p'(x+1)$