HW11 MATH 3520 Fall 2019

#8) $A = \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}$

Find evals of A

$$0 = \det(A - \lambda I) = \det\left(\begin{bmatrix} 5-\lambda & 2 \\ 2 & 5-\lambda \end{bmatrix}\right)$$
$$= (5-\lambda)^2 - 4$$
$$= 25 - 10\lambda + \lambda^2 - 4$$
$$= \lambda^2 - 10\lambda + 21$$
$$= (\lambda - 7)(\lambda - 3)$$

$\Rightarrow$ e-vals are $\lambda = 3, 7$

$\Rightarrow$ by Thm 4.25, A is diagonalizable

evector for $\lambda = 3$

$$\begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3a \\ 3b \end{bmatrix}$$
$$\begin{bmatrix} 5a+2b \\ 2a+5b \end{bmatrix} = \begin{bmatrix} 3a \\ 3b \end{bmatrix}$$
$$\begin{bmatrix} 2a+2b \\ 2a+2b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow a = -b$

$$E_3 = \left\{ \begin{bmatrix} -b \\ b \end{bmatrix} : b \in \mathbb{R} \right\}$$

$\downarrow b = 1$

evector $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

evector for $\lambda = 7$

$$\begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 7a \\ 7b \end{bmatrix}$$
$$\begin{bmatrix} 5a+2b \\ 2a+5b \end{bmatrix} = \begin{bmatrix} 7a \\ 7b \end{bmatrix}$$
$$\begin{bmatrix} -2a+2b \\ 2a-2b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow a = b$

$\Rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an evector

$P^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$

$\Longrightarrow$ By Thm 4.23, $P = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$ works.

Check: $P^{-1}AP = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 7 \end{bmatrix}$, as desired!

#9) $A=\begin{bmatrix}-3 & 4\\ -1 & 1\end{bmatrix}$

eigenvalues of A: $0=\det(A-\lambda I)$

$$=\det\begin{bmatrix}-3-\lambda & 4\\ -1 & 1-\lambda\end{bmatrix}$$

$$=(-3-\lambda)(1-\lambda)-4(-1)$$

$$=-3+3\lambda-\lambda+\lambda^2+4$$

$$=\lambda^2+2\lambda+1$$

$$=(\lambda+1)^2$$

$\Rightarrow$ repeat eigenvalue $\lambda=-1$

$\Rightarrow$ by Thm 4.23, A is not diagonalizable

(a) eigenvalues of $\begin{bmatrix}1 & 3\\ 2 & 0\end{bmatrix}$: $\quad 0=\det\left(\begin{bmatrix}1-\lambda & 3\\ 2 & -\lambda\end{bmatrix}\right)$

$$=\lambda^2-\lambda-6$$

$$=(\lambda-3)(\lambda+2)$$

$\Rightarrow \lambda=3,-2$

evecs for $\lambda=3$

$$\begin{bmatrix}1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}3a\\ 3b\end{bmatrix}$$

$$\begin{bmatrix}a+3b\\ 2a\end{bmatrix}=\begin{bmatrix}3a\\ 3b\end{bmatrix}$$

$$\begin{bmatrix}-2a+3b\\ 2a-3b\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$\Rightarrow a=\frac{3}{2}b$

$\Rightarrow \begin{bmatrix}3/2\\ 1\end{bmatrix}$

evecs for $\lambda=-2$

$$\begin{bmatrix}1 & 3\\ 2 & 0\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}-2a\\ -2b\end{bmatrix}$$

$$\begin{bmatrix}a+3b\\ 2a\end{bmatrix}=\begin{bmatrix}-2a\\ -2b\end{bmatrix}$$

$$\begin{bmatrix}3a+3b\\ 2a+2b\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$\Rightarrow a=-b$

$\Rightarrow \begin{bmatrix}-1\\ 1\end{bmatrix}$

Therefore,

$\vec{x}(t)=c_1e^{3t}\begin{bmatrix}3/2\\ 1\end{bmatrix}+c_2e^{-2t}\begin{bmatrix}-1\\ 1\end{bmatrix}$ is general soln to system of DE's $\vec{x}\,'=\begin{bmatrix}1 & 3\\ 2 & 0\end{bmatrix}\vec{x}$

b) $\vec{x}' = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \vec{x}$

evalues + evecs of the matrix:

eval	3	1	0
evec	$\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Therefore general soln is

$$\vec{x}(t) = c_1 e^{3t} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + c_2 e^{t} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$