Quiz 6 MATH 3504 Spring 2019

$$\begin{cases} x' = -y & \text{(i)} \\ y' = x & \text{(ii)} \\ x(0) = 1,\ y(0) = 0 \end{cases}$$

Soln: From (i), take $\frac{d}{dt}$ to get

$$x'' = -y'$$

Plug in (ii) on right side to get

$$x'' = -x$$

$$\Rightarrow x'' + x = 0$$

To solve, guess $x = e^{\lambda t} \rightarrow x' = \lambda e^{\lambda t}$
$\rightarrow x'' = \lambda^2 e^{\lambda t}$

So we get

$$\lambda^2 e^{\lambda t} + e^{\lambda t} = 0$$

$\downarrow$ divide by $e^{\lambda t}$

$$\lambda^2 + 1 = 0 \rightarrow \lambda = \pm i$$

Therefore, $x(t) = c_1 \cos(t) + c_2 \sin(t)$.

To find $y(t)$, notice that (i) $\rightarrow$ $y = -x'$. So compute

$$x' = -c_1 \sin(t) + c_2 \cos(t)$$

Hence $y = -x' = c_1 \sin(t) - c_2 \cos(t)$

Therefore, solution is

$$\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} c_1 \cos(t) + c_2 \sin(t) \\ c_1 \sin(t) - c_2 \cos(t) \end{pmatrix}$$

To apply initial conditions:

$$\underbrace{\begin{pmatrix}1\\0\end{pmatrix}}_{\text{given}} = \vec{x}(0) = \begin{pmatrix}x(0)\\y(0)\end{pmatrix} = \overbrace{\begin{pmatrix}c_1\cos(0)+c_2\sin(0)\\c_1\sin(0)-c_2\cos(0)\end{pmatrix}}^{\text{computed}} = \begin{pmatrix}c_1\\-c_2\end{pmatrix}$$

So we have $\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}c_1\\-c_2\end{pmatrix} \longrightarrow \begin{matrix}c_1=1\\c_2=0\end{matrix}$

Therefore soln is

$$\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\cos(t)+0\\\sin(t)+0\end{pmatrix} = \begin{pmatrix}\cos(t)\\\sin(t)\end{pmatrix}$$