Quiz 2

Show that $x(t) = \dfrac{e^{2t}-1}{e^{2t}+1}$ solves $\begin{cases} x' = 1-x^2 \\ x(0)=0 \end{cases}$

Soln: Compute

$$x'(t) = \frac{d}{dt}\left[\frac{e^{2t}-1}{e^{2t}+1}\right] \overset{\text{quotient rule}}{=} \frac{(e^{2t}+1)(2e^{2t}) - (e^{2t}-1)(2e^{2t})}{(e^{2t}+1)^2}$$

$$= 2e^{2t}\left[\frac{\cancel{e^{2t}}+1-\cancel{e^{2t}}+1}{(e^{2t}+1)^2}\right]$$

$$= \frac{4e^{2t}}{(e^{2t}+1)^2} = \frac{4e^{2t}}{e^{4t}+2e^{2t}+1}$$

Also

$$x^2 = \left(\frac{e^{2t}-1}{e^{2t}+1}\right)^2 = \frac{(e^{4t}-2e^{2t}+1)}{(e^{4t}+2e^{2t}+1)}$$

Therefore,

$$1-x^2 = 1 - \frac{e^{4t}-2e^{2t}+1}{e^{4t}+2e^{2t}+1}$$

$$= \frac{\cancel{e^{4t}}+2e^{2t}+\cancel{1}-\cancel{e^{4t}}+2e^{2t}-\cancel{1}}{e^{4t}+2e^{2t}+1}$$

$$= \frac{4e^{2t}}{e^{4t}+2e^{2t}+1}$$

Thus we see that $x' = 1-x^2$. Now also notice

$$x(0) = \frac{e^{2(0)}-1}{e^{2(0)}+1} = \frac{1-1}{2} = 0$$