MATH 3504 Quiz 1 (due 16 January)

Given $v(t) = V e^{-\frac{kt}{m}}$, show it solves $\begin{cases} -kv = mv' & (1.1) \\ v(0) = V & (1.2) \end{cases}$

Solution: First we verify (1.2) by plugging in $t=0$:

$$v(0) = V e^{-\frac{k(0)}{m}} = V \underbrace{e^{0}}_{=1} = V$$

To verify (1.1), first compute LHS:

$$-kv = -\left(ke^{-\frac{k}{m}t}\right)V \qquad (\star)$$

and compute RHS:

$$mv' = m\frac{d}{dt}\left[e^{-\frac{kt}{m}}\right]$$

$$= m\left[-\frac{k}{m}e^{-\frac{k}{m}t}\right]$$

$$= -\left(ke^{-\frac{k}{m}t}\right)V \qquad (\star\star)$$

Notice that $(\star)$ and $(\star\star)$ show us

$$-kv = -\left(ke^{-\frac{k}{m}t}\right)V = mv',$$

and therefore we have shown that $v(t) = Ve^{-\frac{kt}{m}}$ solves both (1.1) and (1.2).