MATH 3504 BONUS problem

1. Consider the nonlinear initial value problem

$$\begin{cases} x' = \underbrace{x(y-1)}_{=f(t,x,y)} \\ y' = \underbrace{4 - x^2 - y^2}_{=g(t,x,y)} \\ x(0) = x_0, \quad y(0) = y_0. \end{cases}$$

(note: *the f and g functions do not depend on t (this is typical of the types of nonlinear problems that we solved in class)*)
The Euler method for this system works as it does in the one-dimensional case; we just need to do it twice: for some stepsize h,

$$\begin{cases} x(t_{n+1}) = x(t_n) + hf(t_n, x(t_n), y(t_n)) \\ y(t_{n+1}) = y(t_n) + hg(t_n, x(t_n), y(t_n)) \end{cases}$$

Let the initial condition be defined by your student F-number where

$$\begin{cases} x_0 = -\dfrac{\text{2nd to last digit of your F-number}}{5} \\ y_0 = \dfrac{\text{last digit of your F-number}}{5}. \end{cases}$$

(for example: if your F number is F00000012, then take $x_0 = -\frac{1}{5}$ and $y_0 = \frac{2}{5}$).

1. Use Euler's method with $h = 0.01$ to solve the system for $t \in [0, 20]$. Submit the spreadsheet you used.
2. Plot the solutions $y(t)$ and $x(t)$ as functions of t (i.e. the horizontal axis is the t variable and the vertical axis are the solutions). Submit the picture you get.
3. Plot the phase diagram of this solution (i.e. the horizontal axis is the x variable and the vertical axis is the y variable). Submit the picture you get.
4. What kind of stability appears to be occurring?