HW6 MATH 3504 Spring 2019

p.144 #3

$$\mathcal{L}\{\sin(kt)\} = \frac{1}{2i}\mathcal{L}\{e^{ikt} - e^{-ikt}\}$$

$$= \frac{1}{2i}\left[\mathcal{L}\{e^{ikt}\} - \mathcal{L}\{e^{-ikt}\}\right]$$

$$= \frac{1}{2i}\left[\frac{1}{s-ik} - \frac{1}{s+ik}\right]$$

$$= \frac{1}{2i}\left[\frac{(\cancel{s}+ik) - (\cancel{s}-ik)}{(s-ik)(s+ik)}\right]$$

$$= \frac{\cancel{2i}}{\cancel{2i}}\left[\frac{k}{s^2 - i^2k^2}\right]$$

$$= \frac{k}{s^2+k^2}$$

$\mathcal{L}\{\cos(kt)\}$ is similar.

#8a $\mathcal{L}\{6+5e^{-2t}+te^{3t}\} = 6\mathcal{L}\{1\} + 5\mathcal{L}\{e^{-2t}\} + \mathcal{L}\{te^{3t}\}$

$$= \frac{6}{s} + \frac{5}{s+2} + \frac{1}{(s-3)^2}.$$

using the row in the table which says

$\mathcal{L}\{f(t)e^{at}\} = F(s-a)$

with $f(t) = t$, so $F(s) = \frac{1}{t^2}$

using line of table saying $\mathcal{L}\{f(t)H(t-a)\} = e^{-as}\mathcal{L}\{f(t+a)\}$

#8b $\mathcal{L}\{tH(t-3)\} = e^{-3s}\,\mathcal{L}\{t-3\}$

$$= e^{-3s}\left[\mathcal{L}\{t\} + 3\mathcal{L}\{1\}\right]$$

$$= e^{-3s}\left[\frac{1}{s^2} + \frac{3}{s}\right]$$

#9a $\mathcal{L}^{-1}\left\{\frac{7}{s+2}\right\} = 7\mathcal{L}^{-1}\left\{\frac{1}{s-(-2)}\right\}$

$$= 7e^{-2t}$$

#9d $\mathcal{L}^{-1}\left\{\frac{7}{s}e^{-4s}\right\} = 7\mathcal{L}^{-1}\left\{\frac{1}{s}e^{-4s}\right\}$

$$= 7H(t-4)$$

#9j $\mathcal{L}^{-1}\left\{e^{-2s}\cdot\frac{3}{s^2}\right\} = H(t-2)f(t-2),$

where $f(t)$ is inverse of $F(s) = \frac{3}{s^2}$,
which is $f(t) = 3t$

So,

$$\mathcal{L}^{-1}\left\{e^{-2s}\cdot\frac{3}{s^2}\right\} = H(t-2)\left[3(t-2)\right]$$

p156 **#6a** Solve

$$\begin{cases} x' + 5x = H(t-2) \\ x(0) = 1 \end{cases}$$

Soln: Take $\mathcal{L}$ to get

$$\mathcal{L}\{x'\} + 5\mathcal{L}\{x\} = \mathcal{L}\{H(t-2)\}$$

$$(sX - 1) + 5X = \frac{e^{-2s}}{s}$$

$$(X)(s+5) = \frac{e^{-2s}}{s} + 1$$

$$X = \frac{e^{-2s}}{s(s+5)} + \frac{1}{s+5}$$

Therefore

$$x(t) = \mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s(s+5)}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+5}\right\}$$

$$= H(t-2)\,\mathcal{L}^{-1}\left\{\frac{1}{(s-0)(s-(-5))}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-(-5)}\right\}$$

$$= H(t-2)\left(\frac{1}{0-(-5)}\right)\left(e^{0t} - e^{-5t}\right) + e^{-5t}$$

$$= \frac{H(t-2)}{5}\left(1 - e^{-5(t-2)}\right) + e^{-5t}$$

#6c Soln emailed on 22 Feb led to

$$X(s) = \frac{3}{5}\frac{1}{s-3} + \frac{7}{5}\frac{1}{s-(-2)}$$

So,

$$x(t) = \frac{3}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\} + \frac{7}{5}\mathcal{L}^{-1}\left\{\frac{1}{s-(-2)}\right\}$$

$$= \frac{3}{5}e^{3t} + \frac{7}{5}e^{-2t}$$