HW4 Math 3504 Spring 2019

p.41 #2d) $tx' = -x + t^2$

$$tx' + x = t^2$$

$$x' + \frac{1}{t}x = t$$

$$p(t) = \frac{1}{t} \rightarrow \mu = e^{\int \frac{1}{t}} = e^{\ln(t)} = t$$

Mult by μ

$$(tx)' = t^2$$

$$tx = \frac{t^3}{3} + C \rightarrow \boxed{x(t) = \frac{t^2}{3} + \frac{1}{t}C}$$

#2e) $\theta' = -a\theta + e^{bt}$

$$\theta' + a\theta = e^{bt}$$

$$p(t) = a \rightarrow \mu = e^{\int a} = e^{at}$$

Mult by μ

$$(\theta e^{at})' = e^{(a+b)t}$$

$$\theta e^{at} = \frac{1}{a+b} e^{(a+b)t} + C$$

$$\boxed{\theta(t) = \frac{1}{a+b} e^{bt} + Ce^{-at}}$$

p.90 #1a) $x'' - 4x' + 4x = 0$

Guess: $x(t) = e^{\lambda t} \rightarrow x'(t) = \lambda e^{\lambda t} \rightarrow x''(t) = \lambda^2 e^{\lambda t}$

Plug in to get

$$(\lambda^2 - 4\lambda + 4)\underbrace{e^{\lambda t}}_{\text{never } 0\text{, so divide off}} = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$(\lambda - 2)^2 = 0$$

$$\lambda = 2 \quad \text{(repeated root)}$$

$$\Rightarrow x(t) = c_1 e^{2t} + c_2 t e^{2t}$$

#1b] $x'' - 2x' = 0$

$$\lambda^2 - 2\lambda = 0 \rightarrow \lambda(\lambda - 2) = 0 \rightarrow \lambda = 0, 2$$

$$\Rightarrow \boxed{x(t) = c_1 e^{0t} + c_2 e^{-2t} = c_1 + c_2 e^{2t}}$$

#1c] $\frac{1}{2}x'' + x' + \frac{1}{2}x = 0$

$$\frac{1}{2}\lambda^2 + \lambda + \frac{1}{2} = 0$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$(\lambda+1)^2 = 0 \rightarrow \lambda = -1$ repeated root

$$\boxed{x(t) = c_1 e^{-t} + c_2 t e^{-t}}$$

#1d] $x'' + 4x' + 3x = 0$

$$\lambda^2 + 4\lambda + 3 = 0 \rightarrow \lambda = \frac{-4 \pm \sqrt{16 - 4(3)}}{2} = \frac{-4 \pm 2}{2} \begin{matrix} \xrightarrow{+} -1 \\ \xrightarrow{-} -3 \end{matrix}$$

$$\boxed{x(t) = c_1 e^{-t} + c_2 e^{-3t}}$$

#2 (for #1a)

We got $x(t) = c_1 e^{2t} + c_2 t e^{2t}$

Need to apply IC's $x(0) = 1$ and $x'(0) = 0$

Calculate

$$x'(t) = 2c_1 e^{2t} + c_2 e^{2t} + 2c_2 t e^{2t}$$

So, $$\begin{cases} 1 = x(0) = c_1 & (i) \\ 0 = x'(0) = 2c_1 + c_2 & (ii) \end{cases}$$

$$(i) \rightarrow c_1 = 1 \xrightarrow{(ii)} 0 = 2 + c_2$$

$$\rightarrow \boxed{c_2 = -2}$$

$$\Rightarrow x(t) = -e^{2t} - 2te^{2t}$$

#3) (for #1d)

We got

$$x(t) = c_1 e^{-t} + c_2 e^{-3t}$$

Need to apply IC's $x(0) = -1$ and $x'(0) = 2$.

Compute

$$x'(t) = -c_1 e^{-t} - 3c_2 e^{-3t}$$

Now apply IC's:

$$\begin{cases} -1 = x(0) = c_1 + c_2 & (i) \\ 2 = x'(0) = -c_1 - 3c_2 & (ii) \end{cases}$$

$(i) \rightarrow c_1 = -1 - c_2$

$\xrightarrow{(ii)} 2 = -(-1-c_2) - 3c_2$

$2 = 1 - 2c_2$

$\boxed{p. 94}$ (#1a) $x'' + x' + 4x = 0$

$\lambda^2 + \lambda + 4 = 0$

$\lambda = \frac{-1 \pm \sqrt{1 - 4(4)}}{2} = -\frac{1}{2} \pm \frac{1}{2}\sqrt{15}\, i$

$\Rightarrow x(t) = c_1 e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{15}}{2} t\right) + c_2 e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{15}}{2} t\right)$

(#1b) $x'' - 4x' + 6x = 0$

$\lambda^2 - 4\lambda + 6 = 0 \longrightarrow \lambda = \frac{-(-4) \pm \sqrt{16 - 4(1)(6)}}{2}$

$= 2 \pm \frac{\sqrt{8}}{2} i$

$x(t) = c_1 e^{2t} \cos\left(\frac{\sqrt{8}}{2} t\right) + c_2 e^{2t} \sin\left(\frac{\sqrt{8}}{2} t\right)$

$\begin{array}{r} 24 \\ -16 \\ \hline 8 \end{array}$

#1c] $x'' + 9x = 0$

$\lambda^2 + 9 = 0 \longrightarrow \lambda = \pm 3i$

$\Rightarrow x(t) = c_1 \cos(3t) + c_2 \sin(3t)$

#1d] $x'' - 12x = 0$

$\lambda^2 - 12 = 0$

$\lambda = \pm\sqrt{12} \longrightarrow x(t) = c_1 e^{12t} + c_2 e^{-12t}$

#2 (for #1c)

We got $x(t) = c_1 \cos(3t) + c_2 \sin(3t)$

We need to apply IC's $x(0) = 1$ and $x'(0) = 0$.

Compute

$$x'(t) = -3c_1 \sin(3t) + 3c_2 \cos(3t)$$

Now apply IC's:

$$1 = x(0) = c_1 + 0 \quad \rightarrow c_1 = 1$$

$$0 = x'(0) = 0 + 3c_2 \quad \rightarrow c_2 = 0$$

Therefore we get

$$x(t) = \cos(3t)$$

$\boxed{\text{p.111}}$ (#2a) $x'' + 7x = te^{3t}$

<u>homogeneous:</u> $\lambda^2 + 7 = 0 \to \lambda = \pm\sqrt{7}\, i$

$\Rightarrow x(t) = c_1\cos(\sqrt{7}t) + c_2\sin(\sqrt{7}t)$

<u>particular soln:</u> guess $x_p(t) = (At+B)e^{3t}$

↑(see example 2.16 (e))

$$x_p' = Ae^{3t} + 3Ate^{3t} + 3Be^{3t}$$
$$= (A+3B)e^{3t} + 3Ate^{3t}$$
$$x_p'' = 3(A+3B)e^{3t} + [3Ae^{3t} + 9Ate^{3t}]$$
$$= (6A+9B)e^{3t} + 9Ate^{3t}$$

Plug in to get

$$\left[(6A+9B)\cancel{e^{3t}} + 9At\cancel{e^{3t}}\right] + 7\left((At+B)\cancel{e^{3t}}\right) = t\cancel{e^{3t}}$$

$$(6A+9B+7B) + t(9A+7A) = t$$
$$(6A+16B) + t(16A) = t$$

$$\Rightarrow \begin{cases} 16A = 1 \longrightarrow A = \frac{1}{16} \\ 6A + 16B = 0 \longrightarrow \frac{6}{16} + 16B = 0 \end{cases}$$

$$B = -\frac{6}{16^2}$$

Thus,

$$x(t) = c_1\cos(\sqrt{7}t) + c_2\sin(\sqrt{7}t) + \left(\frac{1}{16}t - \frac{6}{16^2}\right)e^{3t}$$

p. 111 (26) $x'' - x' = 6 + e^{2t}$

homogeneous: $\lambda^2 - \lambda = 0$

$$\lambda(\lambda - 1) = 0$$

$$\lambda = 0, \lambda = 1$$

$$x(t) = c_1 + c_2 e^t$$

particular soln: Solve seperately : $x'' - x' = 6$

AND

$$x'' - x' = e^{2t}$$

$x'' - x' = 6$

need b/c one of ~~homogen solns~~ is constant

$$x_{p_1}(t) = At$$

$$x_{p_1}'(t) = A$$

$$x_{p_1}''(t) = 0$$

↓ plug in

$$0 - A = 6$$

$$\rightarrow A = -6$$

$$\Rightarrow x_{p_1}(t) = -6t$$

$x'' - x' = e^{2t}$

$$x_{p_2}(t) = Ae^{2t}$$

$$x_{p_2}'(t) = 2Ae^{2t}$$

$$x_{p_2}''(t) = 4Ae^{2t}$$

↓ plug in

$$4Ae^{2t} - 2Ae^{2t} = e^{2t}$$

↓

$$2A = 1 \rightarrow A = \frac{1}{2}$$

$$\Rightarrow x_{p_2}(t) = \frac{1}{2}e^{2t}$$

Add particular solns to get final answer:

$$x(t) = c_1 + c_2 e^t \underbrace{-6t}_{x_{p_1}} + \underbrace{\frac{1}{2}e^{2t}}_{x_{p_2}}$$