HW1 MATH 3504 Spring 2019

p.10 #1

a) $x=t^2 \Rightarrow x'=2t$

So, $x'=\frac{2x}{t} \Rightarrow 2t=\frac{2(t^2)}{t}$

plug in

$\Downarrow$

$2t=2t$ TRUE

b) $x=\sqrt{6-t^2} \rightarrow x'=\frac{1}{\cancel{2}\sqrt{6-t^2}}(-\cancel{2}t)=\frac{-t}{\sqrt{6-t^2}}$

So, $x'=-\frac{t}{x} \rightarrow \frac{-t}{\sqrt{6-t^2}}=\frac{-t}{\sqrt{6-t^2}}$ TRUE

plug in

#3 $x=\frac{1}{t} \rightarrow x'=-\frac{1}{t^2}$

$\Downarrow$

$x'=-x^2 \Rightarrow -\frac{1}{t^2} \stackrel{?}{=} -\left(\frac{1}{t}\right)^2$

TRUE $\Rightarrow x=\frac{1}{t}$ is a soln to $x'=-x^2$

$x=\frac{2}{t} \rightarrow x'=\frac{-2}{t^2}$

$x'=-x^2 \Rightarrow -\frac{2}{t^2} \stackrel{?}{=} -\left(\frac{2}{t}\right)^2 \Rightarrow \frac{-2}{t^2}=\frac{-4}{t^2}$

NOT TRUE in general

$x=\frac{1}{t-2} \rightarrow x'=\frac{-1}{(t-2)^2}$

$x'=-x^2 \Rightarrow \frac{-1}{(t-2)^2} \stackrel{?}{=} -\left(\frac{1}{(t-2)}\right)^2 \Rightarrow$ TRUE

**#4** $x_1(t) = e^{-t}\cos(t) \Rightarrow x_1'(t) = -(e^{-t})(\cos(t)+\sin(t))$

$$\Rightarrow x_1''(t) = 2e^{-t}\sin(t)$$

Now compute

$$x_1'' + 2x_1' + 2x_1 = 2e^{-t}\sin(t) + 2\left[-e^{-t}(\cos(t)+\sin(t))\right] + 2\left[e^{-t}\cos(t)\right]$$

$$= 0,$$

showing $x_1$ solves $x_1'' + 2x_1' + 2x_1 = 0$

Similar calculation shows $x_2 = e^{-t}\sin(t)$ also solves it.

**#7** $x(t) = t^m \rightarrow x' = mt^{m-1}$

Plug in to $2tx' = x$ to get

$$2t(mt^{m-1}) = t^m$$

$$t^m(2m-1) = 0$$

$t^m = 0 \quad 2m-1=0$

$\downarrow \quad m = \frac{1}{2}$

no soln for $m$

Let $P(t)$ denote population at year $t$, $\Rightarrow$ units of $P'(t)$ are $\frac{\text{people}}{\text{year}}$

#16 Let $t=0$ correspond to year: 1999.

We are told population $\underbrace{P}_{\text{population}}(\underbrace{0}_{\text{in year 1999}}) = 6{,}000{,}000{,}000$ people

UNI

"$\underbrace{\text{increasing}}_{\text{derivative}}$ $212{,}000 \frac{\text{people}}{\text{day}}$" $\rightsquigarrow P'(0) = 212000 \frac{\text{people}}{\text{day}}$

$$= 212000 \frac{\text{people}}{\text{day}} \left(\frac{365 \text{ days}}{\text{year}}\right)$$

$$= 77380000 \frac{\text{people}}{\text{year}}$$

Growth eqt: $\frac{dP}{dt} = rP$

$$\downarrow$$

$$\underbrace{\frac{P'}{P}}_{} = r$$

$$\boxed{\text{recall: } = \frac{d}{dt}\ln(P)}$$

$$\Rightarrow \frac{d}{dt}\ln(P) = r$$

$$\downarrow \int \ldots dt$$

$$\ln(P) = rt + C$$

$$\downarrow$$

$$\boxed{P = C_1 e^{rt}} ; \; C_1 = e^C$$

Find $C_1$

$$\underbrace{6000000000}_{\text{given}} = P(0) = \underbrace{C_1 e^0}_{=C_1} \Rightarrow \boxed{P(t) = 6000000000 e^{rt}}$$

Growth rate: find r. Use only other known info:

$$P'(0) = 77380000$$

To use it, calculate

$$p'(t) = 6000000000 r e^{rt}, \text{ hence}$$

$$77380000 = p'(0) = 6000000000 r \underbrace{e^{0}}_{=1}$$

Therefore,

$$r = \frac{77380000}{6000000000} \approx 0.0128$$

Therefore model suggests

$$P(t) = 6000000000 e^{0.0128t}$$

Year 2050: $t=0 \to 1999$

$t=1 \to 2000$

$t=2 \to 2001$

$\vdots$

$t=51 \to 2050$

So the model predicts in 2050, population would be

$$P(51) = 6000000000 e^{0.0128(51)}$$

$$\approx 11{,}525{,}471{,}156$$

(note: model predicts 2019 pop to be:

$$P(20) = 6000000000 e^{0.0128(20)}$$

$$= 7{,}750{,}516{,}368$$

which seems roughly correct!!)