Homework 13

Euler's method says to approximate the solution to $x' = f(t, x), \quad x(t_0) =_0$ by the following recurrence relation

$$\begin{cases} x(t_0) = x_0 \\ x(t_{n+1}) = x(t_n) + hf(t_n, x(t_n)). \end{cases}$$

The equation we are approximating is

$$\begin{cases} x' = \underbrace{e^{-\sin(x)}}_{=f(t,x)} \\ x(0) = -1 \end{cases}$$

In all the following problems, the initial condition gives us that $x(t_0) = -1$.

1. With $h = 0.5$ we have $t_0 = 0$, $t_1 = 0.5$, and $t_2 = 1$. Using the recurrence relation, we see that

$$\begin{aligned} \underbrace{x(t_1)}_{=x(0.5)} &= x(t_0) + he^{-\sin(x(t_0))} \\ &= -1 + 0.5(e^{-\sin(-1)}) \\ &= -1 + 0.5(2.319) \\ &= 0.1595 \end{aligned}$$

and

$$\begin{aligned} \underbrace{x(t_2)}_{x(1)} &= x(t_1) + he^{-\sin(x(t_1))} \\ &= 0.1595 + 0.5e^{-\sin(0.1595)} \\ &= 0.5860 \end{aligned}$$

From this we have shown that $x(1) \approx 0.5860$.

2. With $h = 0.25$ we have $t_0 = 0$, $t_1 = 0.25$, $t_2 = 0.5$, $t_3 = 0.75$, and $t_4 = 1$. Therefore we approximate:

$$\begin{aligned} \underbrace{x(t_1)}_{=x(0.25)} &= x(t_0) + he^{-\sin(x(t_0))} \\ &= -1 + 0.25e^{-\sin(-1)} \\ &= -0.4200, \end{aligned}$$

$$\begin{aligned} \underbrace{x(t_2)}_{=x(0.5)} &= -0.4200 + 0.25e^{-\sin(-0.4200)} \\ &= -0.0441, \end{aligned}$$

$$\begin{aligned} \underbrace{x(t_3)}_{=x(0.75)} &= -0.0441 + 0.25e^{-\sin(-0.0441)} \\ &= 0.2171, \end{aligned}$$

and finally,

$$\begin{aligned} \underbrace{x(t_4)}_{x(1)} &= 0.2171 + 0.25e^{-\sin(0.2171))} \\ &= 0.4186. \end{aligned}$$