p. 257

#1a $\begin{cases} x' = x - y \\ y' = 2x(-1+y) \end{cases}$

equilibrium

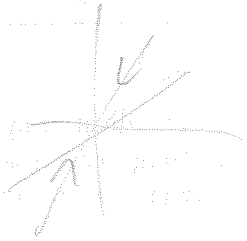
$\begin{cases} 0 = x - y \longrightarrow x = y \\ 0 = 2x(-1+y) \longrightarrow \text{plug into (i)} \end{cases}$

$0 = 2x(-1+x)$

$x = 0 \qquad -1+x = 0$

$x = 1$

$(0,0) \qquad (1,1)$



Jacobian

$$J = \begin{pmatrix} f_x & f_y \\ g_x & g_y \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 2y & -2x \end{pmatrix}$$

Linearizations

At (0,0)

$$\vec{x}' = \begin{pmatrix} 1 & -1 \\ -2 & 0 \end{pmatrix} \vec{x}$$

e-vals. 2 and −1

saddle

At (1,1)

$$\vec{x}' = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix} \vec{x}$$

evals $-1, 0$

x-nullclines

$0 = x - y$

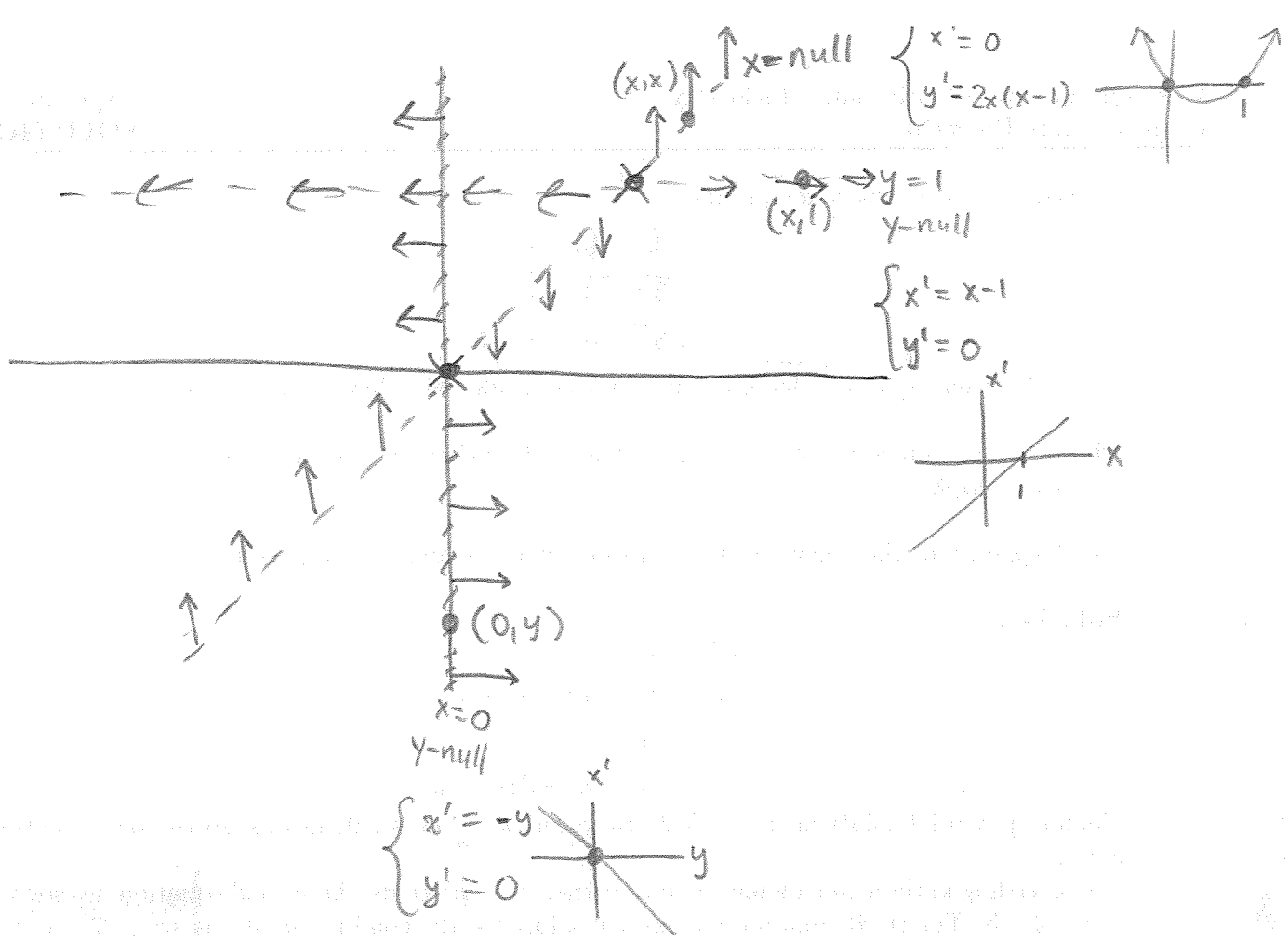
$y = x$

y-nullclines

$0 = 2x(-1+y)$

$x = 0 \qquad -1+y = 0$

$y = 1$


(x,x)
x=null
x'=0
y'=2x(x-1)
1
(x,1)
y=1
y-null
x'=x-1
y'=0
x'
x
1
(0,y)
x=0
y-null
x'
x'=-y
y'=0
y

#1b)

$$\begin{cases} x' = x - xy^2 \\ y' = x + y \end{cases}$$

equilibrium

$$\begin{cases} 0 = x - xy^2 & (i) \\ 0 = x + y & (ii) \end{cases}$$

(i) $\rightarrow$ $0 = x(1 - y^2)$

$x = 0$ or $1 - y^2 = 0$

$1 - y^2 = 0$: $y = 1$ or $y = -1$

$x = 0$ and (ii) $\rightarrow$ $0 = 0 + y$ $\rightarrow$ $y = 0$ $\rightarrow$ $(0,0)$

$y = 1$ and (ii) $\rightarrow$ $0 = x + 1$ $\rightarrow$ $x = -1$ $\rightarrow$ $(-1,1)$

$y = -1$ and (ii) $\rightarrow$ $0 = x - 1$ $\rightarrow$ $x = 1$ $\rightarrow$ $(1,-1)$

Jacobian: $J = \begin{pmatrix} 1 - y^2 & -2xy \\ 1 & 1 \end{pmatrix}$

Linearized systems

at $(0,0)$

$\vec{x}' = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \vec{x}$

evals $\rightarrow$ 1 (repeated)

$\downarrow$

unstable

at $(-1,1)$

$\vec{x}' = \begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix} \vec{x}$

at $(1,-1)$

$\vec{x}' = \begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix} \vec{x}$

evals $2, -1$

$\downarrow$

saddle (unstable)

x-nullclines

$0 = x - xy^2$

$0 = x(1-y^2)$

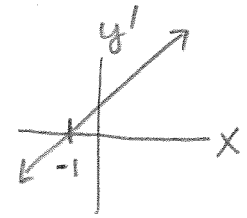
$x = 0 \quad y = \pm 1$

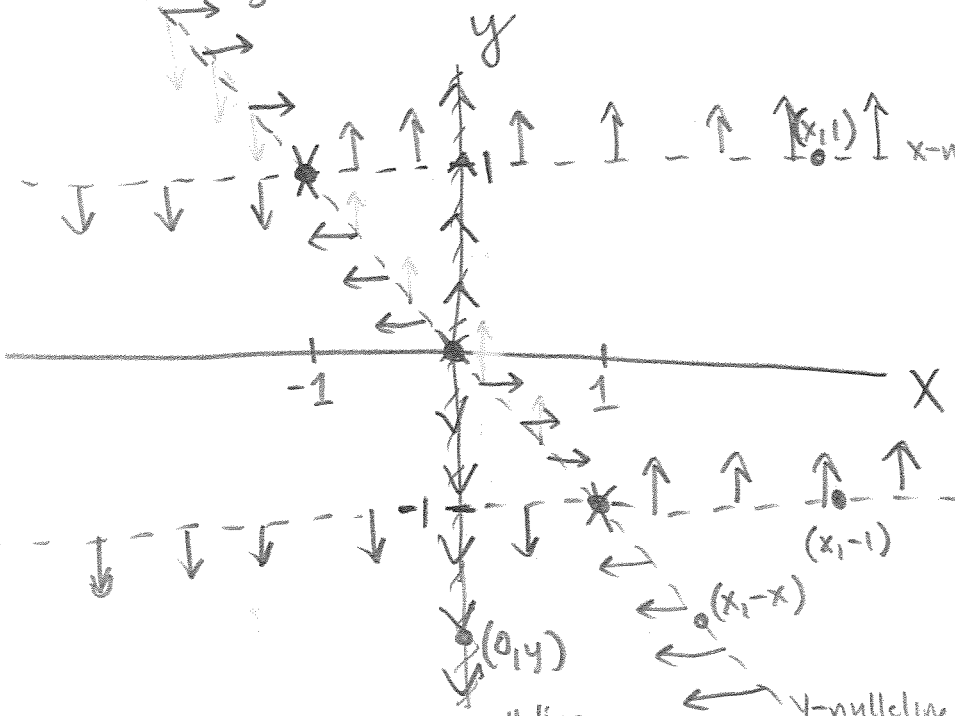
y-nullclines

$0 = x + y$

$\downarrow$

$y = -x$



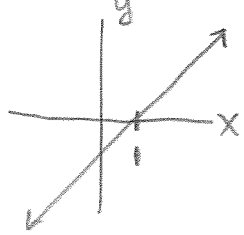


x-nullcline
$y = 1$

$\begin{cases} x' = 0 \\ y' = x + 1 \end{cases}$

x-nullcline
$y = -1$

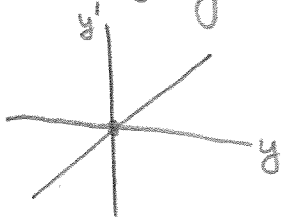
$\begin{cases} x' = 0 \\ y' = x - 1 \end{cases}$



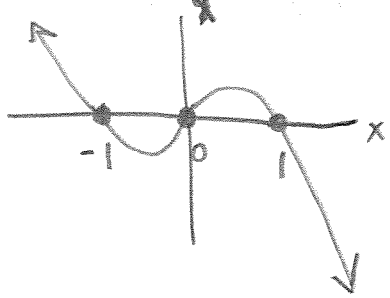
y-nullcline
$y = -x$

x-nullcline
$x = 0$

$\begin{cases} x' = 0 \\ y' = y \end{cases}$



$\begin{cases} x' = x(1-x^2) \\ y' = 0 \end{cases}$



#1 d

$$\begin{cases} x' = -x + xy \\ y' = -2y(1-4x) \end{cases}$$

equilibria

$$\begin{cases} 0 = -x + xy & \text{(i)} \\ 0 = -2y(1-4x) & \text{(ii)} \end{cases}$$

(i) $\rightarrow$ $0 = x(-1+y)$

$x = 0$ or $-1+y = 0$, $y = 1$

(ii)

$0 = -2y(1-0)$, $y = 0$ $\Rightarrow$ $(0,0)$

$-2(1-4x) = 0$, $x = \frac{1}{4}$ $\Rightarrow$ $(\frac{1}{4}, 1)$

Jacobian: $J = \begin{pmatrix} -1+y & x \\ 8y & -2+8x \end{pmatrix}$

Linearized System

at $(0,0)$

$\vec{x}' = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \vec{x}$

evals $\rightarrow$ $-1, -2$

$\downarrow$

stable node

at $(\frac{1}{4}, 1)$

$\vec{x}' = \begin{pmatrix} 0 & 1/4 \\ 8 & 0 \end{pmatrix} \vec{x}$

evals $\rightarrow$ $\lambda = -\sqrt{2}, \lambda = \sqrt{2}$

$\downarrow$

saddle

x-nullclines

$0 = -x + xy$

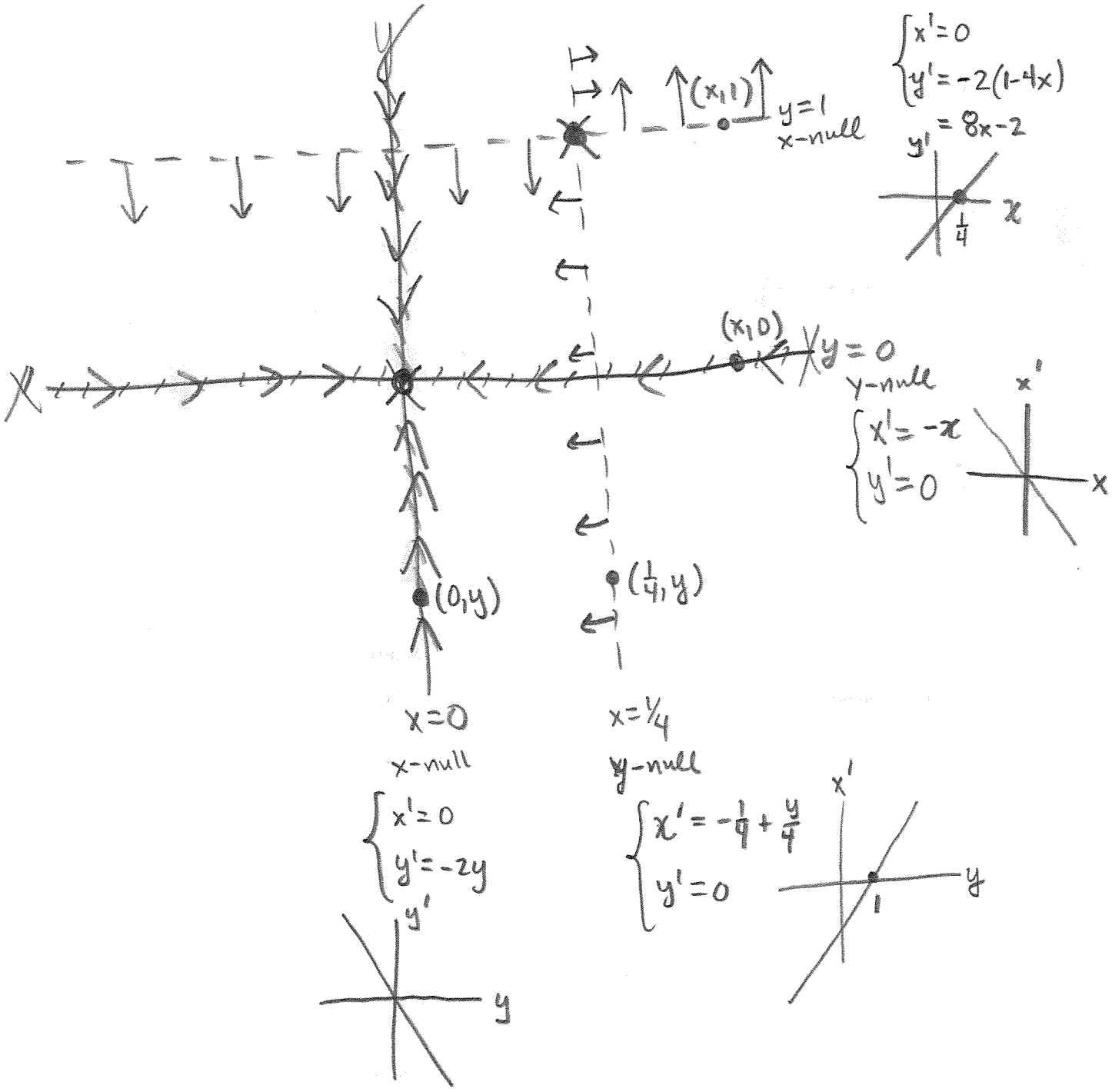
$0 = x(-1+y)$

$x=0,\ y=1$

y-nullclines

$0 = -2y(1-4x)$

$y=0 \quad x=\frac{1}{4}$



$\begin{cases} x' = 0 \\ y' = -2(1-4x) \end{cases}$

$y' = 8x - 2$

$\begin{cases} x' = -x \\ y' = 0 \end{cases}$

$\begin{cases} x' = 0 \\ y' = -2y \end{cases}$

$\begin{cases} x' = -\frac{1}{4} + \frac{y}{4} \\ y' = 0 \end{cases}$