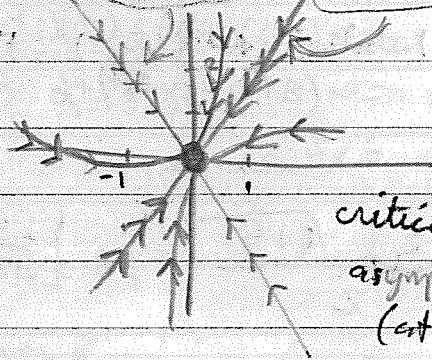


Homework 10 MATH 3504 Spring 2019

p. 218 #1a) eigenpairs:  $\lambda = -1, \vec{v} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$   
 $\lambda = -2, \vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

⇒ General solution

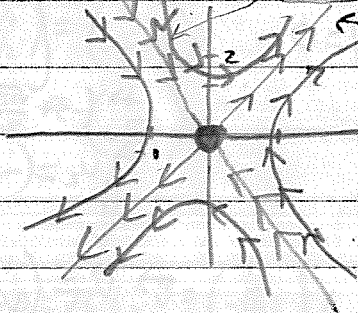
$$\vec{x} = c_1 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$$



critical pt at (0,0)  
asymptotically stable  
(attractor)

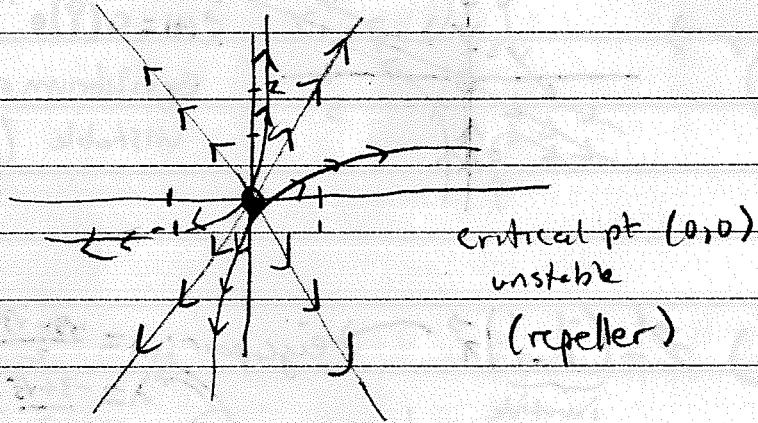
#1b) eigenpairs:  $\lambda = -1, \vec{v} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$   
 $\lambda = 2, \vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

⇒ gen soln:  $\vec{x} = c_1 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t}$

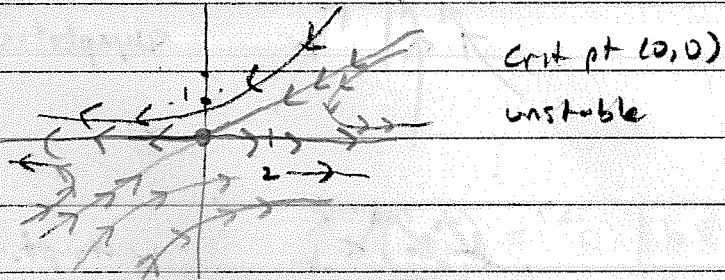


crit. pt. at (0,0)  
unstable

#1c) eigenpair:  $\lambda=1, \vec{v} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$  gen soln  
 $\lambda=2, \vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \rightarrow \vec{x}(t) = c_1 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^t + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t}$



#1d) eigenpair:  $\lambda=-3, \vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$  gen soln  
 $\lambda=2, \vec{v} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \rightarrow \vec{x} = c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} -1 \\ 0 \end{pmatrix} e^{2t}$

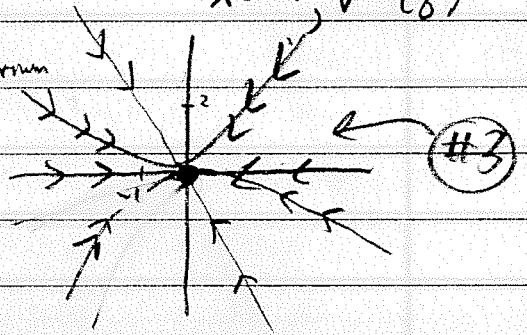


#2a) Solve  $\vec{x}' = \begin{pmatrix} -1 & 1 \\ 0 & -3 \end{pmatrix} \vec{x}$  gen soln  
 $\vec{x} = c_1 \begin{pmatrix} -1 \\ 2 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-3t}$

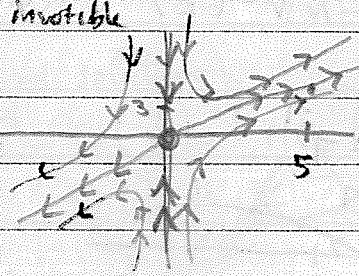
Soln:  $A = \begin{pmatrix} -1 & 1 \\ 0 & -3 \end{pmatrix}$  has eigenpair  $\lambda=-3, \vec{v} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$   
 $\lambda=-1, \vec{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\det A \neq 0$   
 $\hookrightarrow$  invertible  
 $\hookrightarrow$  isolated equilibrium at (0,0)

crit pt at (0,0)  
asymptotically stable



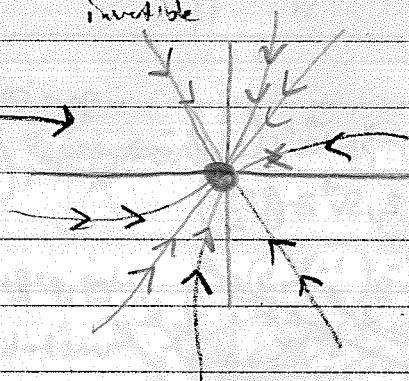
#2b)  $\vec{x}' = \begin{pmatrix} 1 & 0 \\ 3 & -4 \end{pmatrix} \vec{x}$  eigenpair:  $\lambda = -4, \vec{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$   
 $\lambda = 1, \vec{v} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$



#3

gen soln  
 $\vec{x}(t) = c_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-4t} + c_2 \begin{pmatrix} 5 \\ 3 \end{pmatrix} e^t$   
 equilibrium at  $(0,0)$   
 unstable (saddle pt)

#2c)  $\vec{x}' = \begin{pmatrix} -1 & 1 \\ 1 & -2 \end{pmatrix} \vec{x}$  eigenpair:  $\lambda = \frac{-3-\sqrt{5}}{2}, \vec{v} = \begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{pmatrix}$   
 $\lambda = \frac{-3+\sqrt{5}}{2}, \vec{v} = \begin{pmatrix} \frac{1+\sqrt{5}}{2} \\ 1 \end{pmatrix}$



#3

both  $\ominus$  gen soln  
 $\vec{x} = c_1 \begin{pmatrix} \frac{1-\sqrt{5}}{2} \\ 1 \end{pmatrix} e^{\frac{-3-\sqrt{5}}{2}t} + c_2 \begin{pmatrix} \frac{1+\sqrt{5}}{2} \\ 1 \end{pmatrix} e^{\frac{-3+\sqrt{5}}{2}t}$   
 equilibrium at  $(0,0)$   
 asymptotically stable