Propositional Consequences

From...	Consequence	Name
P	$\neg\neg P$	double negation
$\neg\neg P$	P	double negation
F, G	$F \wedge G$	$\wedge$-introduction
F	$F \vee G$	$\vee$-introduction
$F \wedge G$	$G \wedge F$	$\wedge$-commutativity
$F \vee G$	$G \vee F$	$\vee$-commutativity
$F \wedge G$	F	$\wedge$-elimination
$F, F \to G$	G	$\to$-elimination
$F \vee G$	$\neg(\neg F \wedge \neg G)$	DeMorgan
$\neg(\neg F \wedge \neg G)$	$F \vee G$	DeMorgan
$F \wedge G$	$\neg(\neg F \vee \neg G)$	DeMorgan
$\neg(\neg F \vee \neg G)$	$F \wedge G$	DeMorgan
$F \leftrightarrow G$	$(F \to G) \wedge (G \to F)$	Biconditional
$(F \to G) \wedge (G \to F)$	$F \leftrightarrow G$	Biconditional
$P \to Q$	$\neg Q \to \neg P$	contraposition
$\neg P, P \vee Q$	Q	modus tollendo ponens (m.t.p.)
$P \to Q$, $Q \to R$	$P \to R$	syllogism
$P \to Q$	$\neg P \vee Q$	equivalence of disjunction and implication
$\neg P \vee Q$	$P \to Q$	equivalence of disjunction and implication
$(P \vee R) \wedge (P \vee \neg R)$	P	proof by cases

Rule of conditional proof: if G is a consequence of $P_1, \ldots, P_n$ and an additional premise F, then $F \to G$ is a consequence of $P_1, \ldots, P_n$

Proof by contradiction: if the contradiction $G \wedge \neg G$ is a consequence of premises $P_1, \ldots, P_n$ and an additional premise F, then $\neg F$ is a consequence of the premises $P_1, \ldots, P_n$

Replacing double negations inside of a formula: If $\neg\neg P$ appears as a subformula of a formula F, then the formula G, where G is the same as F except that $\neg\neg P$ is replaced with P, may be concluded.

Substitution of propositionally equivalent statement: If a statement Q is propositionally equivalent to a statement P, then any instance of Q inside any statement may be replaced with P.

$\forall$-elimination: The formula $\forall x Px$ can be replaced with $P\alpha$, where α is ***any*** constant or variable.

$\forall$-introduction: The formula $\forall x Px$ can be concluded whenever Pv was previously concluded, **only when** v is a variable.

$\exists$-elimination: From $\exists x Px$, one can conclude $P\alpha$, where α is a constant *that has not previously been used in the proof.*

$\exists$-introduction: If $P\alpha$ is known, where α is a *constant or a variable*, then $\exists x Px$ can be concluded.