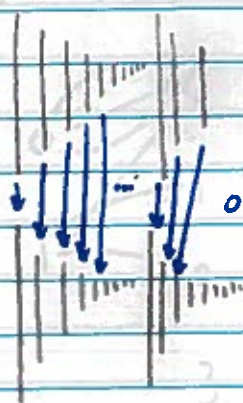
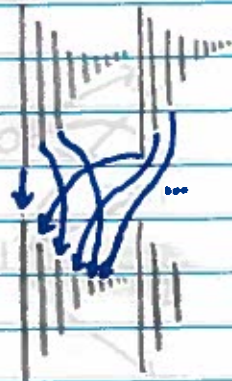


HW9 MATH 251D Spring 2019

①



one-to-one



one-to-one

②

- a) $\emptyset, \{0\}, \{1\}, \{2\}, \{3\}, \{0,1\}, \{0,2\}, \{0,3\}$
 $\{1,2\}, \{1,3\}, \{2,3\}, \{0,1,2\}, \{0,1,3\}, \{0,2,3\}, \{1,2,3\}, \{0,1,2,3\}$

b) not cofinal crossed out in blue

c) cardinalities in red

d) smallest cardinality $\rightarrow 1$

e) $\text{cf}(w+1) = 1$ because any subset containing w is cofinal, so $\{w\}$ is cofinal

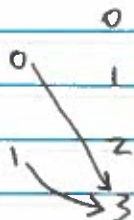
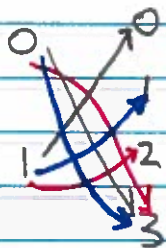
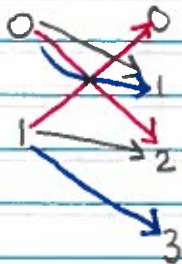
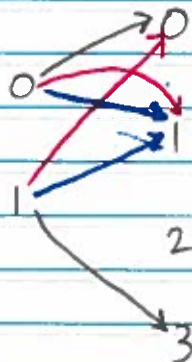
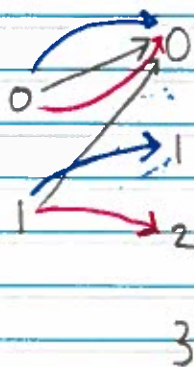
③

$$\begin{aligned} \text{a) } 1 \otimes 2 &= \text{card}(\{0\} \times \{0\} \cup \{0,1\} \times \{1\}) \\ &= \text{card}(\{(0,0)\} \cup \{(0,1), (1,1)\}) \\ &= \text{card}(\{(0,0), (0,1), (1,1)\}) \\ &= 3 = \text{card}(\omega) = 3 \end{aligned}$$

$$\begin{aligned} \text{b) } 3 \otimes 3 &= \text{card}(\{0,1,2\} \times \{0,1,2\}) \\ &= \text{card}(\{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2)\}) \\ &= 9 \end{aligned}$$

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c) $4^2 = \text{card}({}^2_4)$



There are 16 functions described, so

$$4^2 = \text{card}({}^2_4) = 16$$