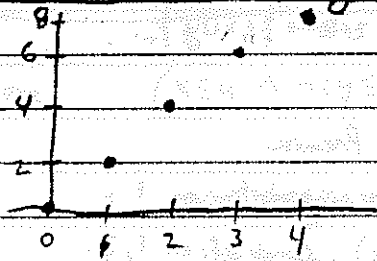


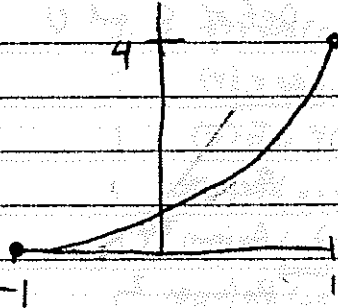
HW8 MATH 2510 Spring 2019

①



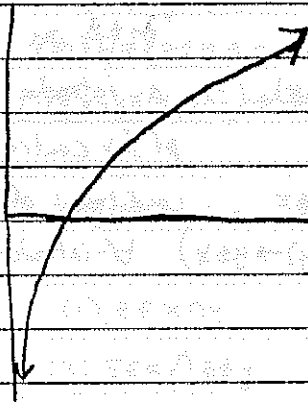
yes one-to-one

②



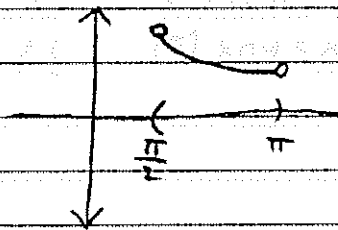
yes one-to-one

③



yes one-to-one

④



yes one-to-one

$$\begin{aligned} \left(\frac{d^2}{dx^2} + x\right)(ax^2+bx+c) &= \frac{d}{dx}(2ax+b) + (ax^3+bx^2+cx) \\ &= 2a + ax^3+bx^2+cx \\ &= ax^3+bx^2+(x+2a) \end{aligned}$$

5) Yes one-to-one. If $p=ax^2+bx+c$, then

$$D(p) = \left(\frac{d^2}{dx^2} + x\right)p$$

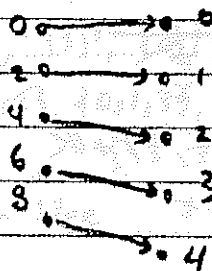
$$= \frac{d^2}{dx^2}(ax^2+bx+c) + x(ax^2+bx+c)$$

$$= \frac{d}{dx}(2ax+b) + (ax^3+bx^2+cx)$$

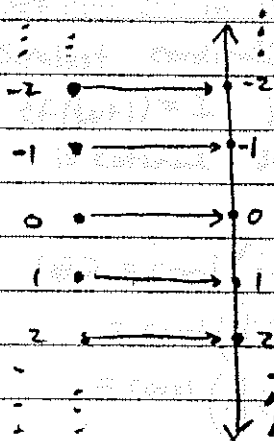
$$= ax^3+bx^2+cx+2a$$

It is not possible to pick a, b, c so that we get two different input polynomials w/ same output.

6) Use $\begin{cases} f: 2\mathbb{N} \rightarrow \mathbb{N} \\ f(n) = \frac{n}{2} \end{cases}$



7)



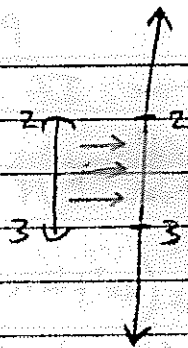
$$\begin{cases} f: \mathbb{Z} \rightarrow \mathbb{R} \\ f(x) = x \end{cases}$$

$$b) 3 \otimes 3 = \text{card}(\{0,1,2\} \times \{0,1,2\})$$

$$= \text{card}(\{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), (2,2)\})$$

$$= 9$$

$$8) \begin{cases} f: (-2, 3) \rightarrow \mathbb{R} \\ f(x) = x \end{cases}$$



$$9) \text{card}\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \leq \text{card}(\mathbb{R})$$

$$\text{card}(\mathbb{R}) \leq \text{card}\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

