HW7 MATH 2510 Spring 2019

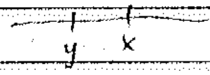
① Prove $(x \subseteq y \wedge \neg(y \subseteq x)) \longrightarrow \exists z\,(z \in x \wedge \neg(z \in y))$

1	(1) $x \subseteq y \wedge \neg(y \subseteq x)$	Premise
1	(2) $\forall z(z\in x \to z \in y) \wedge \neg \forall w(w \in y \to w \in x)$	Def of $\subseteq$
1	(3) $\forall z(z \in x \to z \in y)$	$\wedge$-elim on 2
1	(4) $\neg \forall w(w \in y \to w \in x)$	$\wedge$-elim on 2
1	(5) $\exists w\, \neg(w \in y \to w \in x)$	negation of $\forall$ on 4
1	(6) $\neg(\alpha \in y \to \alpha \in x)$	$\exists$-elim — $w=\alpha$ on 5
1	(7) $\neg(\neg(\alpha \in y) \vee \alpha \in x)$	equiv of disj + impl on 6
1	(8) $\neg\neg(\alpha \in y) \wedge \neg(\alpha \in x)$	DeMorgan on 7
1	(9) $\alpha \in y \wedge \neg(\alpha \in x)$	Double Neg on 8
1	(10) $\exists z(z \in y \wedge \neg(z \in x))$	$\exists$-introduction on 9
{ }	(11) $(x \subseteq y \wedge \neg(y \subseteq x)) \to \exists z(z \in y \wedge \neg(z \in x))$	conditional pf. 1,10

② Goal: prove $x \cap y \subseteq x$, i.e. $z \in x \cap y \to z \in x$

1	(1) $z \in x \cap y$	Premise
1	(2) $z \in x \wedge z \in y$	Def of $\cap$
1	(3) $z \in x$	$\wedge$-elim on 2
{ }	(4) $z \in x \cap y \to z \in x$	Conditional pf on 1,3
{ }	(5) $x \cap y \subseteq x$	def of $\subseteq$ and $\cap$

(3) goal: prove $x \subseteq x \cup y$ (i.e. $z \in x \to z \in x \cup y$)



1	(1) $z \in x$	Premise
1	(2) $z \in x \vee z \in y$	$\vee$-introduction on 1
{ }	(3) $z \in x \to (z \in x \vee z \in y)$	Cond pf on 1,2
{ }	(4) $x \subseteq x \cup y$	Def of $\subseteq$ and $\cup$

(4) a)

variables

1	(1) $\neg(x \leq y)$	Premise
T4	(2) $\forall x \forall y (x \leq y \vee y \leq x)$	Axiom T4
T4	(3) $x \leq y \vee y \leq x$	$\forall$elim on 2
4	(4) $\neg(y \leq x)$	Premise
4, T4	(5) $x \leq y$	mtp on 3,4
1, 4, T4	(6) $x \leq y \wedge \neg(x \leq y)$	$\wedge$-introduction on 1,5
1, T4	(7) $y \leq x$	pf by contradiction on 4,6
T4	(8) $\neg(x \leq y) \to y \leq x$	conditional pf on 1,7
T4	(9) $\forall x \forall y (\neg(x \leq y) \to y \leq x)$	$\forall$-introduction on 8