Homework 7 – MATH 2510 Spring 2019

1. Prove the following theorem of naive set theory:

$$\Big((x \subseteq y) \wedge \neg(y \subseteq x)\Big) \rightarrow \exists z(z \in y \wedge \neg(z \in x))$$

2. Define the intersection notation $x \cap y$: we write $z \in x \cap y$ to mean $z \in x \wedge z \in y$. Prove that $x \cap y \subseteq x$.

3. Define the union notation $x \cup y$: we write $z \in x \cup y$ to mean $z \in x \vee z \in y$. Prove that $x \subseteq x \cup y$.

4. Consider a new theory called "total order theory" which has a two-place predicate "$\leq$" (meaning it compares two elements) and the following axioms:

Axiom T1	$(\forall x)(\forall y)(\forall z)(x \leq y \wedge y \leq z \rightarrow x \leq z)$
Axiom T2	$(\forall x)(x \leq x)$
Axiom T3	$(\forall x)(\forall y)(x \leq y \wedge y \leq x \rightarrow x = y$
Axiom T4	$(\forall x)(\forall y)(x \leq y \vee y \leq x)$

Prove the following in total order theory or give an interpretation to show it is not true:

a.) $(\forall x)(\forall y)(\neg(x \leq y) \rightarrow y \leq x)$

b.) $x \leq y \wedge \neg(x \leq z) \rightarrow \neg(y \leq z)$