HW 6 MATH 2510 Spring 2019

1a

1	(1)	$x=y \wedge (y=z \wedge z=w)$	Premise
1	(2)	$x=y$	$\wedge$-elim on 1
1	(3)	$y=z \wedge z=w$	$\wedge$-elim on 1
1	(4)	$y=z$	$\wedge$-elim on 3
1	(5)	$z=w$	$\wedge$-elim on 3
1	(6)	$y=w$	"=" sub on 4,5
1	(7)	$x=w$	"=" sub on 2,6
{ }	(8)	$(x=y \wedge (x=z \wedge z=w)) \to x=w$	cond proof on 1,7

empty

1b

1	(1)	$(x=y \wedge \neg(y=z))$	Premise
1	(2)	$x=y$	$\wedge$-elim on 1
1	(3)	$\neg(y=z)$	$\wedge$-elim on 1
1	(4)	$\neg(x=z)$	"="-subst. on 2,3
{ }	(5)	$(x=y \wedge \neg(y=z)) \to \neg(x=z)$	cond proof 1,4

empty

2a

Axiom 7	(1)	$\forall x \forall y (x \cdot Sy = (x \cdot y) + x)$	Axiom 7
Ax 7	(2)	$\forall y (S0 \cdot Sy = (S0 \cdot y) + S0)$	$\forall$-elim on 1, $x=S0$
Ax 7	(3)	$S0 \cdot S0 = (S0 \cdot 0) + S0$	$\forall$elim on 2, $y=0$
Ax 6	(4)	$\forall x (x \cdot 0 = 0)$	Axiom 6
Ax 6	(5)	$S0 \cdot 0 = 0$	$\forall$-elim on 4, $x=S0$
Ax 6,7	(6)	$S0 \cdot S0 = 0 + S0$	"="-substit. on 3,5
Ax 4	(7)	$\forall x (x + 0 = x)$	Axiom 4
Ax 4	(8)	$S0 + 0 = S0$	$\forall$-elim on 7, $x=S0$
Ax 8	(9)	$\forall x \forall y (x+y = y+x)$	Axiom 8
Ax 8	(10)	$\forall y (S0 + y = y + S0)$	$\forall$elim on 9, $x=S0$

Ax 8	(11) $S0+0=0+S0$	$\forall$elim on 10, $y=0$
Ax 6, 7, 8	(12) $S0 \cdot S0 = S0+0$	"=" substit. on 11, 6
Ax 4, 6, 7, 8	(13) $S0 \cdot S0 = S0$	"=" substit on 12, 8

2b

Ax 7	(1) $\forall x \forall y (x \cdot Sy = (x \cdot y)+x)$	Axiom 7
Ax 7	(2) $\forall y (S0 \cdot Sy = (S0 \cdot y)+S0)$	$\forall$-elim on 1, $x=S0$
Ax 7	(3) $S0 \cdot SS0 = (S0 \cdot S0)+S0$	$\forall$-elim on 2, $y=S0$
Ax 4, 6, 7, 8	(4) $S0 \cdot S0 = S0$	Problem 2a
Ax 4, 6, 7, 8	(5) $S0 \cdot SS0 = S0+S0$	"="-subst. 3, 4

2c

Ax 5	(1) $\forall x \forall y (x+Sy = S(x+y))$	Axiom 5
Ax 5	(2) $\forall y (SS0+Sy = S(SS0+y))$	$\forall$-elim on 1, $x=SS0$
Ax 5	(3) $SS0+SSS0 = S(SS0+SS0)$	$\forall$-elim on 2, $y=SS0$
Ax 4, 5	(4) $SS0+SS0=SSSS0$	Proved in class
Ax 4, 5	(5) $SS0+SSS0=SSSSS0$	"=" substit 3, 4

3

1	(1) $\forall x\, Rxa$	Premise
Ax G1	(2) $\forall x \forall y (Rxy \to Ryx)$	Axiom G1
1	(3) Rba	$\forall$-elim on 1, $x=b$
1	(4) Rca	$\forall$-elim on 1, $x=c$
Ax G1	(5) $\forall y (Rby \to Ryb)$	$\forall$-elim on 2, $x=b$
Ax G1	(6) $Rba \to Rab$	$\forall$elim on 5, $y=a$
Ax G1	(7) $\forall y (Rcy \to Ryc)$	$\forall$-elim on 2, $x=c$
Ax G1	(8) $Rca \to Rac$	$\forall$-elim on 7, $y=a$
1, Ax G1	(9) Rab	$\to$-elim on 3, 6
1, Ax G1	(10) Rac	$\to$-elim on 4, 8
1, Ax G1	(11) $Rab \wedge Rac$	$\wedge$-introduction 9, 10
1, Ax G1	(12) $\exists y (Rab \wedge Ryc)$	$\exists$-intro on 11
1, Ax G1	(13) $\exists x \exists y (Rxb \wedge Ryc)$	$\exists$-intro on 12