Homework 6 – MATH 2510 Spring 2019

1. Prove the following in pure identity theory.

   (a) $(x = y) \wedge ((y = z) \wedge (z = w)) \rightarrow x = w$

   (b) $((x = y) \wedge \neg(y = z)) \rightarrow \neg(x = z)$

2. Prove the following in first order arithmetic.

   a. $S0 \cdot S0 = S0$

   b. $S0 \cdot SS0 = S0 + S0$

   c. $SS0 + SSS0 = SSSSS0$

3. Consider a new theory called "graph theory" which obeys the following two axioms:
   G1 ("Symmetric"): $\forall x \forall y (Rxy \rightarrow Ryx)$
   G2 ("Anti-reflexive"): $\forall x \neg R(x, x)$
   We think of Rxy as standing in for "x is directly connected to y", so **G1** is saying that "if x is directly connected to y, then y is directly connected to x" and **G2** is saying "nothing is directly connected to itself" ("no loops").

   Assume an extra premise $\forall x R(x, a)$ (where a is a constant). Prove from the axioms of graph theory and this additional premise that
   $\exists x \exists y (R(x, b) \wedge R(y, c))$