

HWS MATH 2510 Spring 2019

- ①
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|-----|-------------------------------------|------------------------------|
| 1 | (1) $\forall x (Ax \rightarrow Bx)$ | Premise |
| 2 | (2) $\forall x (Bx \rightarrow Cx)$ | Premise |
| 1 | (3) $Ay \rightarrow By$ | \forall -elim on (1) |
| 2 | (4) $By \rightarrow Cy$ | \forall -elim on (2) |
| 1,2 | (5) $Ay \rightarrow Cy$ | Syllogism on 3,4 |
| 1,2 | (6) $\forall x (Ax \rightarrow Cx)$ | \forall -introduction on 5 |

- ②
- | | | |
|-----|-------------------------------------|-----------------------------------|
| 1 | (1) $\forall x (Bx \rightarrow Cx)$ | Premise |
| 2 | (2) $\exists x (Ax \wedge Bx)$ | Premise |
| 2 | (3) $Ax \wedge Bx$ | \exists -elimination on 2 |
| 1 | (4) $Bx \rightarrow Cx$ | \forall -elimination on 1 |
| 2 | (5) Bx | \wedge -elimination on 3 |
| 1,2 | (6) Cx | \rightarrow -elimination on 4,5 |
| 2 | (7) Ax | \wedge -elimination on 3 |
| 1,2 | (8) $Ax \wedge Cx$ | \wedge -introduction on 6,7 |
| 1,2 | (9) $\exists x (Ax \wedge Cx)$ | \exists -introduction on 8 |

- ③
- | | | |
|-----|---|----------------------------|
| 1 | (1) $\forall x ((Ax \vee Bx) \rightarrow Cx)$ | Premise |
| 2 | (2) $\exists y (Ay \wedge Dy)$ | Premise |
| 2 | (3) $Ax \wedge Dx$ | \exists -elim on 2 |
| 1 | (4) $(Ax \vee Bx) \rightarrow Cx$ | \forall -elim on 1 |
| 2 | (5) Ax | \wedge -elim on 3 |
| 2 | (6) $Ax \vee Bx$ | \vee -introduction on 5 |
| 1,2 | (7) Cx | \rightarrow -elim on 4,6 |
| 1,2 | (8) $\exists y Cy$ | \exists -intro on 7 |

- (4)
- | | | | |
|-----|-----|---|----------------------------|
| 1 | (1) | $(\forall x Ax) \rightarrow (\exists x Bx)$ | Premise |
| 2 | (2) | $\forall x (\neg Bx)$ | Premise |
| 1 | (3) | $\neg(\exists x Bx) \rightarrow \neg(\forall x Ax)$ | Contrapositive on 1 |
| 2 | (4) | $\neg \exists x \neg(Bx)$ | neg of \exists on 2 |
| 2 | (5) | $\neg \exists x (Bx)$ | double neg on 4 |
| 1,2 | (6) | $\neg(\forall x Ax)$ | \rightarrow -elim on 3,5 |
| 1,2 | (7) | $\exists x \neg Ax$ | neg of \forall on 6 |

- (5)
- | | | | |
|-----|-----|--|-----------------------|
| 1 | (1) | $(\exists x \neg Ax) \vee (\exists x \neg Bx)$ | Premise |
| 2 | (2) | $\forall x Bx$ | Premise |
| 2 | (3) | $\neg(\exists x \neg Bx)$ | neg of \exists on 2 |
| 1,2 | (4) | $\exists x \neg Ax$ | m.t.p. on 1,3 |
| 1,2 | (5) | $\neg \forall x (\neg \neg Ax)$ | neg of \forall on 4 |
| 1,2 | (6) | $\neg \forall x Ax$ | double neg on 5 |

$\exists x Bx \wedge \forall x$

- (8)
- | | | | | |
|--------|------|---|------------------------------------|-------------------|
| 1 | (1) | $\forall x ((Bx \vee Px) \rightarrow Lx)$ | Premise | $P \rightarrow Q$ |
| 2 | (2) | $\neg \exists x (Lx \wedge Gx)$ | Premise | $\neg P \vee Q$ |
| 2 | (3) | $\forall x \neg(Lx \wedge Gx)$ | neg of \exists on 2 | |
| 2 | (4) | $\neg(Ly \wedge Gy)$ | \forall -elim on 3 | |
| 2 | (5) | $\neg Ly \vee \neg Gy$ | De Morgan on 4 | |
| 2 | (6) | $\neg Gy \vee \neg Ly$ | \vee -commutative 5 | |
| 2 | (7) | $Gy \rightarrow \neg Ly$ | equiv disj & impl 6 | |
| 1 | (8) | $(By \vee Py) \rightarrow Ly$ | \forall -elim on 1 | |
| 1 | (9) | $\neg Ly \rightarrow \neg(By \vee Py)$ | Contrapositive of 8 | |
| 10 | (10) | By | premise | |
| 10 | (11) | $By \vee Py$ | \vee -introduction on 10 | |
| 1,10 | (12) | Ly | \rightarrow -elimination on 8,11 | |
| 1,2,10 | (13) | $\neg Gy$ | m.t.p on 6,12 | |

1105 p. 102, 1125, 1126M, 1127M

1.2 (14) $B \rightarrow \neg G$ Conditional proof on 10, 13

1.2 (15) $\neg B \vee \neg G$ Equivalence of \vee and \rightarrow on 14

1.2 (16) $\neg(B \wedge G)$ De Morgan on 15

1.2 (17) $\forall x \neg(Bx \wedge Gx)$ \forall -introduction on 16

1.2 (18) $\neg \exists x (Bx \wedge Gx)$ negation of \exists on 17

(25)

$(\forall x \neg(Bx \wedge Gx)) \rightarrow \neg \exists x (Bx \wedge Gx)$
 Assume $\neg \exists x (Bx \wedge Gx)$
 Assume $\exists x (Bx \wedge Gx)$
 \exists -elimination: $Ba \wedge Ga$
 \wedge -elimination: Ba, Ga
 \exists -introduction: $\exists x (Bx \wedge Gx)$
 Contradiction: $\exists x (Bx \wedge Gx) \wedge \neg \exists x (Bx \wedge Gx)$
 \neg -elimination: $\neg \exists x (Bx \wedge Gx)$

(26)

$(\exists x (Bx \wedge Gx)) \rightarrow \neg \forall x \neg(Bx \wedge Gx)$
 Assume $\forall x \neg(Bx \wedge Gx)$
 Assume $\exists x (Bx \wedge Gx)$
 \exists -elimination: $Ba \wedge Ga$
 \wedge -elimination: Ba, Ga
 \forall -elimination: $\neg(Ba \wedge Ga)$
 Contradiction: $Ba \wedge Ga \wedge \neg(Ba \wedge Ga)$
 \neg -elimination: $\neg \forall x \neg(Bx \wedge Gx)$

(27)

(1) $\forall x Rxx$ Premise
 (2) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (1)
 (3) $\forall x (Rxx \rightarrow Rxx) \rightarrow \forall x Rxx$ \rightarrow -introduction (2)
 (4) $\forall x (Rxx \rightarrow Rxx) \rightarrow \forall x Rxx$ \forall -introduction (3)
 (5) $\forall x Rxx$ \forall -elimination (4)
 (6) Raa \forall -elimination (5)
 (7) $Raa \rightarrow Raa$ \rightarrow -introduction (6)
 (8) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (7)
 (9) $\forall x Rxx$ \forall -elimination (8)
 (10) Raa \forall -elimination (9)
 (11) $Raa \rightarrow Raa$ \rightarrow -introduction (10)
 (12) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (11)
 (13) $\forall x Rxx$ \forall -elimination (12)
 (14) Raa \forall -elimination (13)
 (15) $Raa \rightarrow Raa$ \rightarrow -introduction (14)
 (16) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (15)
 (17) $\forall x Rxx$ \forall -elimination (16)
 (18) Raa \forall -elimination (17)
 (19) $Raa \rightarrow Raa$ \rightarrow -introduction (18)
 (20) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (19)
 (21) $\forall x Rxx$ \forall -elimination (20)
 (22) Raa \forall -elimination (21)
 (23) $Raa \rightarrow Raa$ \rightarrow -introduction (22)
 (24) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (23)
 (25) $\forall x Rxx$ \forall -elimination (24)
 (26) Raa \forall -elimination (25)
 (27) $Raa \rightarrow Raa$ \rightarrow -introduction (26)
 (28) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (27)
 (29) $\forall x Rxx$ \forall -elimination (28)
 (30) Raa \forall -elimination (29)
 (31) $Raa \rightarrow Raa$ \rightarrow -introduction (30)
 (32) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (31)
 (33) $\forall x Rxx$ \forall -elimination (32)
 (34) Raa \forall -elimination (33)
 (35) $Raa \rightarrow Raa$ \rightarrow -introduction (34)
 (36) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (35)
 (37) $\forall x Rxx$ \forall -elimination (36)
 (38) Raa \forall -elimination (37)
 (39) $Raa \rightarrow Raa$ \rightarrow -introduction (38)
 (40) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (39)
 (41) $\forall x Rxx$ \forall -elimination (40)
 (42) Raa \forall -elimination (41)
 (43) $Raa \rightarrow Raa$ \rightarrow -introduction (42)
 (44) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (43)
 (45) $\forall x Rxx$ \forall -elimination (44)
 (46) Raa \forall -elimination (45)
 (47) $Raa \rightarrow Raa$ \rightarrow -introduction (46)
 (48) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (47)
 (49) $\forall x Rxx$ \forall -elimination (48)
 (50) Raa \forall -elimination (49)
 (51) $Raa \rightarrow Raa$ \rightarrow -introduction (50)
 (52) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (51)
 (53) $\forall x Rxx$ \forall -elimination (52)
 (54) Raa \forall -elimination (53)
 (55) $Raa \rightarrow Raa$ \rightarrow -introduction (54)
 (56) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (55)
 (57) $\forall x Rxx$ \forall -elimination (56)
 (58) Raa \forall -elimination (57)
 (59) $Raa \rightarrow Raa$ \rightarrow -introduction (58)
 (60) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (59)
 (61) $\forall x Rxx$ \forall -elimination (60)
 (62) Raa \forall -elimination (61)
 (63) $Raa \rightarrow Raa$ \rightarrow -introduction (62)
 (64) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (63)
 (65) $\forall x Rxx$ \forall -elimination (64)
 (66) Raa \forall -elimination (65)
 (67) $Raa \rightarrow Raa$ \rightarrow -introduction (66)
 (68) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (67)
 (69) $\forall x Rxx$ \forall -elimination (68)
 (70) Raa \forall -elimination (69)
 (71) $Raa \rightarrow Raa$ \rightarrow -introduction (70)
 (72) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (71)
 (73) $\forall x Rxx$ \forall -elimination (72)
 (74) Raa \forall -elimination (73)
 (75) $Raa \rightarrow Raa$ \rightarrow -introduction (74)
 (76) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (75)
 (77) $\forall x Rxx$ \forall -elimination (76)
 (78) Raa \forall -elimination (77)
 (79) $Raa \rightarrow Raa$ \rightarrow -introduction (78)
 (80) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (79)
 (81) $\forall x Rxx$ \forall -elimination (80)
 (82) Raa \forall -elimination (81)
 (83) $Raa \rightarrow Raa$ \rightarrow -introduction (82)
 (84) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (83)
 (85) $\forall x Rxx$ \forall -elimination (84)
 (86) Raa \forall -elimination (85)
 (87) $Raa \rightarrow Raa$ \rightarrow -introduction (86)
 (88) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (87)
 (89) $\forall x Rxx$ \forall -elimination (88)
 (90) Raa \forall -elimination (89)
 (91) $Raa \rightarrow Raa$ \rightarrow -introduction (90)
 (92) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (91)
 (93) $\forall x Rxx$ \forall -elimination (92)
 (94) Raa \forall -elimination (93)
 (95) $Raa \rightarrow Raa$ \rightarrow -introduction (94)
 (96) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (95)
 (97) $\forall x Rxx$ \forall -elimination (96)
 (98) Raa \forall -elimination (97)
 (99) $Raa \rightarrow Raa$ \rightarrow -introduction (98)
 (100) $\forall x (Rxx \rightarrow Rxx)$ \forall -introduction (99)