HW5 MATH 2510 Spring 2019

①

1	(1) $\forall x\,(Ax \to Bx)$	Premise
2	(2) $\forall x\,(Bx \to Cx)$	Premise
1	(3) $Ay \to By$	$\forall$-elim on (1)
2	(4) $By \to Cy$	$\forall$-elim on (2)
1,2	(5) $Ay \to Cy$	Syllogism on 3,4
1,2	(6) $\forall x\,(Ax \to Cx)$	$\forall$-introduction on 5

②

1	(1) $\forall x\,(Bx \to Cx)$	Premise
2	(2) $\exists x\,(Ax \wedge Bx)$	Premise
2	(3) $A\alpha \wedge B\alpha$	$\exists$-elimination on 2
1	(4) $B\alpha \to C\alpha$	$\forall$-elimination on 1
2	(5) $B\alpha$	$\wedge$-elimination on 3
1,2	(6) $C\alpha$	$\to$-elimination on 4,5
2	(7) $A\alpha$	$\wedge$-elimination on 3
1,2	(8) $A\alpha \wedge C\alpha$	$\wedge$-introduction on 6,7
1,2	(9) $\exists x\,(Ax \wedge Cx)$	$\exists$-introduction on 8

③

1	(1) $\forall x\,((Ax \vee Bx) \to Cx)$	Premise
2	(2) $\exists y\,(Ay \wedge Dy)$	Premise
2	(3) $A\alpha \wedge D\alpha$	$\exists$-elim on 2
1	(4) $(A\alpha \vee B\alpha) \to C\alpha$	$\forall$-elim on 1
2	(5) $A\alpha$	$\wedge$-elim on 3
2	(6) $A\alpha \vee B\alpha$	$\vee$-introduction on 5
1,2	(7) $C\alpha$	$\to$-elim on 4,6
1,2	(8) $\exists y\, Cy$	$\exists$-intro on 7

(4)

1	(1)	$(\forall x Ax) \to (\exists x Bx)$	Premise
2	(2)	$\forall x (\neg Bx)$	Premise
1	(3)	$\neg(\exists x Bx) \to \neg(\forall x Ax)$	Contrapositive on 1
2	(4)	$\neg \exists x \neg(\neg Bx)$	neg of $\exists$ on 2
2	(5)	$\neg \exists x (Bx)$	double neg on 4
1,2	(6)	$\neg(\forall x Ax)$	$\to$-elim on 3,5
1,2	(7)	$\exists x \neg Ax$	neg of $\forall$ on 6

(5)

1	(1)	$(\exists x \neg Ax) \vee (\exists x \neg Bx)$	Premise
2	(2)	$\forall x Bx$	Premise
2	(3)	$\neg(\exists x \neg Bx)$	neg of $\exists$ on 2
1,2	(4)	$\exists x \neg Ax$	m.t.p. on 1,3
1,2	(5)	$\neg \forall x (\neg\neg Ax)$	neg of $\forall$ on 4
1,2	(6)	$\neg \forall x Ax$	double neg on 5

$\neg \exists x\, Bx \wedge Gx$

$\forall x\, \neg Bx \vee \neg Gx$

(8)

$P \to Q$

$\neg P \vee Q$

1	(1)	$\forall x ((Bx \vee Px) \to Lx)$	Premise
2	(2)	$\neg \exists x (Lx \wedge Gx)$	Premise
2	(3)	$\forall x \neg(Lx \wedge Gx)$	neg of $\exists$ on 2.
2	(4)	$\neg(Ly \wedge Gy)$	$\forall$-elim on 3
2	(5)	$\neg Ly \vee \neg Gy$	DeMorgan on 4
2	(6)	$\neg Gy \vee \neg Ly$	$\vee$-commutative 5
2	(7)	$Gy \to \neg Ly$	equiv disj & impl 6
1	(8)	$(By \vee Py) \to Ly$	$\forall$-elim on 1
1	(9)	$\neg Ly \to \neg(By \vee Py)$	Contrapositive of 8
10	(10)	By	premise
10	(11)	$By \vee Py$	$\vee$-introduction on 10
1,10	(12)	Ly	$\to$-elimination on 8,11
1,2,10	(13)	$\neg Gy$	m.t.p on 6,12

1,2	(14) $By \to \neg Gy$	Conditional proof on 10,13
1,2	(15) $\neg By \vee \neg Gy$	equivalence of $\vee$ and $\to$ on 14
1,2	(16) $\neg(By \wedge Gy)$	DeMorgan on 15
1,2	(17) $\forall x\, \neg(Bx \wedge Gx)$	$\forall$-introduction on 16
1,2	(18) $\neg\exists x\,(Bx \wedge Gx)$	negation of $\exists$ on 17