Homework 5 – MATH 2510 Spring 2019

1. Prove $\forall x(Ax \to Cx)$ from the premises $\forall x(Ax \to Bx)$ and $\forall x(Bx \to Cx)$.
2. Prove $\exists x(Ax \wedge Cx)$ from the premises $\forall x(Bx \to Cx)$ and $\exists x(Ax \wedge Bx)$.
3. $\exists y(Cy)$ from the premises $\forall x((Ax \vee Bx) \to Cx)$ and $\exists y(Ay \wedge Dy)$.
4. Prove $\exists x \neg Ax$ from the premises $(\forall x Ax) \to (\exists x Bx)$ and $\forall x \neg Bx$.
5. Prove $\neg \forall x(Ax)$ from the premises $(\exists x \neg Ax) \vee (\exists x \neg Bx)$ and $\forall x Bx$.
6. Prove $\forall x Bx$ from the premises $(\forall x(Ax \wedge Bx)) \vee (\forall x(Cx \wedge Dx))$ and $\neg \forall x Dx$.
7. Prove $\neg \exists x Ax$ from the premises $(\exists x(Ax \vee Bx)) \to \forall x Cx$ and $\exists x \neg Cx$.
8. Translate the statements into symbols and write the proof.
   a. All beans and peas are legumes. There are no legumes in the garden. Therefore, there are no beans in the garden.
   b. There are cucumbers in the garden. If there are any cucumbers in the garden, then there are some pumpkins in the garden. All pumpkins are vegetables. Therefore, there are some vegetables in the garden.
9. Translate the argument into symbols and write the proof:

   "Every philosophical empiricist admires Hume. Some philosophical idealists like no one who admires Hume. Therefore, some philosophical idealists like no philosophical empiricist."