Homework 2 – MATH 2510 Spring 2019

1. Determine whether the statement is a tautology, a contradiction, or neither by constructing a truth table.

   a. $(\neg P \vee Q) \vee (P \wedge Q)$

   Solution:

P	Q	$\neg P$	$\neg P \vee Q$	$P \wedge Q$	$(\neg P \vee Q) \vee (P \wedge Q)$
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	T	F	T
F	F	T	T	F	T

   Therefore, the formula $(\neg P \vee Q) \vee (P \wedge Q)$ is neither a tautology nor a contradiction.

   b. $(\neg P \vee \neg Q) \vee (P \wedge Q)$

   Solution:

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$P \wedge Q$	$(\neg P \vee \neg Q) \vee (P \wedge Q)$
T	T	F	F	F	T	T
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

   Therefore, the formula $(\neg P \vee \neg Q) \vee (P \wedge Q)$ is a tautology.

   c. $P \to (P \vee Q)$

   Solution:

P	Q	$P \vee Q$	$P \to (P \vee Q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

   Therefore $P \to (P \vee Q)$ is a tautology.

   d. $Q \leftrightarrow (P \wedge Q)$

   Solution:

P	Q	$P \wedge Q$	$Q \leftrightarrow (P \wedge Q)$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

   Therefore, the formula $Q \leftrightarrow (P \wedge Q)$ is neither a tautology nor a contradiction.

2. Determine whether the two statements are propositionally equivalent or not by constructing a truth table.

   a. P and $P \to P$

   Solution:

P	$P \to P$	$P \leftrightarrow (P \to P)$
T	T	T
F	T	T

   Since $P \leftrightarrow (P \to P)$ is a tautology, we may conclude that $P \to P$ is propositionally equivalent to P.

   b. $P \to Q$ and $\neg P \to \neg Q$

   Solution:

P	Q	$\neg P$	$\neg Q$	$P \to Q$	$\neg P \to \neg Q$	$(P \to Q) \leftrightarrow (\neg P \to \neg Q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Since $(P \to Q) \leftrightarrow (\neg P \to \neg Q)$ is not a tautology, we conclude that $P \to Q$ is **not** propositionally equivalent to $\neg P \to \neg Q$.

c. P and $P \vee \neg P$

Solution:

P	Q	$\neg P$	$P \vee \neg P$	$P \leftrightarrow (P \vee \neg P)$
T	T	F	T	T
T	F	F	T	T
F	T	T	T	F
F	F	T	T	F

Since $P \leftrightarrow (P \vee \neg P)$ is not a tautology, we conclude that P is **not** propositionally equivalent to $P \vee \neg P$.

3. Determine whether or not the second statement is a propositional consequence of the first statement by constructing a truth table.

a. $P \wedge (P \to Q)$ and Q

Solution:

P	Q	$P \to Q$	$P \wedge (P \to Q)$	$(P \wedge (P \to Q)) \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since $(P \wedge (P \to Q)) \to Q$ is a tautology, we conclude that Q is a propositional consequence of $(P \wedge (P \to Q))$.

b. $\neg P \wedge (P \to Q)$ and P

Solution:

P	Q	$\neg P$	$P \to Q$	$\neg P \wedge (P \to Q)$	$(\neg P \wedge (P \to Q)) \to P$
T	T	F	T	F	T
T	F	F	F	F	T
F	T	T	T	T	F
F	F	T	T	T	F

Since $(\neg P \wedge (P \to Q)) \to P$ is not a tautology, we conclude that P is **not** a propositional consequence of $\neg P \wedge (P \to Q)$.

4. Show that D is a propositional consequence of $(D \vee O) \wedge \neg O$.

Solution:

D	O	$\neg O$	$D \vee O$	$(D \vee O) \wedge \neg O$	$((D \vee O) \wedge \neg O) \to D$
T	T	F	T	F	T
T	F	T	T	T	T
F	T	F	T	F	T
F	F	T	F	F	T

5. Show that $\neg C$ is a propositional consequence of $(C \to \neg D) \wedge D)$.
Solution:

C	D	$\neg D$	$C \to \neg D$	$(C \to \neg D) \wedge D$	$((C \to \neg D) \wedge D) \to D$
T	T	F	F	F	T
T	F	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T