Homework 1 – MATH 2510 Spring 2019

Definitions

Recall...

Element of: To write $a \in A$ means that "a" is an object inside of the set A.
Subset: $A \subset B$ means for all $a \in A$, it is also true that $a \in B$.
Cartesian product: $A \times B = \{(a, b) : a \in A, b \in B\}$
Relation: A relation R on a set X is a subset of $X \times X$, i.e. $R \subseteq X \times X$.
Function: A function is a relation R where given any two $(a, b) \in R$ and $(c, d) \in R$, it's always the case that $a \neq c$.
AND ($\wedge$): $P \wedge Q$ is true only when both P is true and Q is true
OR ($\vee$): $P \vee Q$ is true whenever either P is true, Q is true, or both P and Q are true
IMPLIES ($\longrightarrow$): $P \to Q$ is true when both P and Q are true or anytime when P is false
IF AND ONLY IF ($\longleftrightarrow$): $P \leftrightarrow Q$ is true if both P and Q are true or if both P and Q are false
NEGATION ($\neg$): $\neg P$ is true only when P is false

Problems

1. Let $X = \{1, 3, 4\}$, $Y = \{2, 4, 5\}$, and $Z = \{1, 2, 3, 5, 7\}$.
   (a) Is $2 \in X$? Is $2 \in Y$? Is $2 \in Z$?
   (b) Is $X \subseteq Y$? Is $X \subseteq Z$? Is $Y \subseteq Z$? Is $Z \subseteq X$? Is $Z \subseteq Y$?
   (c) What is $X \cup Y$? What is $X \cup Z$? What is $Y \cup Z$? What is $X \cup Y \cup Z$?
   (d) What is $X \cap Y$? What is $X \cap Z$? What is $Y \cap Z$? What is $X \cap Y \cap Z$?
   (e) What is $X \times Y$?
2. Let $X = \{1, 3, 8\}$. Let $R \subset X \times X$ be a relation given by $R = \{(1, 1), (3, 1), (8, 1)\}$. Is R a function? Why or why not?
3. What does the symbol $\emptyset$ denote in set theory?
4. Is the following string of symbols a formula of propositional logic? Why or why not?
$$(A \vee \neg) \vee (A \wedge (B \wedge \neg A)).$$
5. Is the following string of symbols a formula of propositional logic? Why or why not?
$$(A \wedge (\neg B)) \wedge ((\neg A) \wedge B)$$
6. Let P, Q, and R be statements. Given that P is true, Q is false, and R is also false, state whether the following formulas are true or false and explain your answer.
   (a) $P \to (Q \vee R)$
   (b) $R \to (Q \wedge P)$
   (c) $\neg P \longleftrightarrow Q$
   (d) $(P \vee R) \leftrightarrow (Q \vee R)$
   (e) $P \wedge (Q \wedge (R \vee P))$
7. Make truth tables for the formulas (a), (c), and (e) of Problem 6.