① (1)
$$\begin{array}{ccccccccccccccccc} S & 0 & \cdot & S & S & 0 & = & (& S & 0 & \cdot & S & 0 &) & + & S & 0 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 2 & 3 & 4 & 2 & 2 & 3 & 5 & 6 & 2 & 3 & 4 & 2 & 3 & 7 & 8 & 2 & 3 \end{array}$$

(2)
$$\begin{array}{cccccccc} S & 0 & \cdot & S & 0 & = & S & 0 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 2 & 3 & 4 & 2 & 3 & 5 & 2 & 3 \end{array}$$

(3)
$$\begin{array}{cccccccccccc} S & 0 & \cdot & S & S & 0 & = & S & 0 & + & S & 0. \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 2 & 3 & 4 & 2 & 2 & 3 & 5 & 2 & 3 & 8 & 2 & 3 \end{array}$$

Gödel #'s

line 1:

$$g_1 = 2^2\, 3^3\, 5^4\, 7^2\, 11^2\, 13^3\, 17^5\, 19^6\, 23^2\, 29^3\, 31^4\, 37^2\, 41^3\, 43^7\, 47^8\, 53^2\, 59^3$$

= an 80-digit # that I'm not going to write

$$g_2 = 2^2\, 3^3\, 5^4\, 7^2\, 11^2\, 13^3\, 17^5\, 19^2\, 23^3\, 29^8\, 31^2\, 37^3$$

$$g_3 = 2^2\, 3^3\, 5^4\, 7^2\, 11^2\, 13^3\, 17^5\, 19^2\, 23^3\, 29^8\, 31^2\, 37^3$$

②

$$2^{2^2 3^3 5^4 7^2 11^2 13^3 17^5 19^6 23^2 29^3 31^4 37^2 41^3 43^7 47^8 53^2 59^3} \cdot 3^{2^2 3^3 5^4 7^2 11^2 13^3 17^5 19^2 23^3 29^8 31^2 37^3} \cdot 5^{2^2 3^3 5^4 7^2 11^2 13^3 17^5 19^2 23^3 29^8 31^2 37^3}$$

= HUGE number

(3) $\mathcal{P}(I) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{3\}, \{0,1\}, \{0,2\}, \{0,3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{0,1,2\}, \{0,1,3\}, \{0,2,3\}, \{1,2,3\}, \{0,1,2,3\}\}$

(4) No because it fails axiom F3 ~ $\{0,1\} \in \mathcal{F}$ and $\{0,1\} \subseteq \{0,1,2\} \in \mathcal{P}(I)$, but $\{0,1,2\} \notin \mathcal{F}$

(5) yes

$\boxed{F1}$ ~ $\emptyset \notin \mathcal{F}$

$\boxed{F2}$ — all nonempty intersections of elements of $\mathcal{F}$ are in $\mathcal{F}$

$\boxed{F3}$ ~ all supersets of elements in $\mathcal{F}$ are in $\mathcal{F}$