Homework 10 – MATH 2510 Spring 2019

1. Recall the definition of the Ackermann function from the notes. Compute $\mathrm{Ack}(2, 3)$.

2. Consider a function defined recursively by the formula

$$\begin{cases} x(0) = 2 \\ x(n+1) = 3x(n) \end{cases}$$

What is $x(4)$? What is $x(5)$? In general, what is the (non-recursive) formula for $x(n)$?

Consider the theory of four-point geometry defined by the following axioms:

Axiom 1	There exist exactly four points.
Axiom 2	Each two distinct points have exactly one line that contains both of them.
Axiom 3	Each line is exactly on two points.

3. Draw a picture ("model") of four-point geometry.

4. Show that the axioms of four-point geometry are all independent from each other.

Consider six-line geometry defined by the following axioms:

Axiom 1	There exist exactly six lines.
Axiom 2	Each two distinct lines have exactly one point on both of them.
Axiom 3	Each point is on exactly two lines.

5. Draw a model of six-line geometry.

6. Show that the axioms of six-line geometry are all independent from each other.

Consider Fano's geometry defined by the following five axioms:

Axiom 1	There exists at least one line.
Axiom 2	Every line has exactly 3 points on it.
Axiom 3	Not all points are on the same line.
Axiom 4	For two distinct points, there exists exactly one line on both of them.
Axiom 5	Each two lines have at least one point on both of them.

7. Draw a model for Fano geometry (hint: letting your "lines" be curved can be helpful!)