§5.2
84, 85, 88, 90, 94, 95, 98, 102, 103, 104

138, 139, 141, 142, 146, 152, 153, 155, 156, 158, 160, 161, 165, 169, 170

HW9 MATH 2502 Spring 2019

§5.2 #90 $1 + \frac{e}{\pi} + \frac{e^2}{\pi^2} + \frac{e^3}{\pi^3} + \cdots$

$$= \sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n \leftarrow \text{geometric w/ } r = \frac{e}{\pi} \approx \frac{2.7}{3.14} < 1$$

Since geometric w/ $|r| < 1$, it converges.

(in fact: converges to value $\frac{1}{1 - \frac{e}{\pi}} = \frac{\pi}{\pi - e}$)

#94 $\begin{cases} a_1 = 2 \\ \frac{a_n}{a_{n+1}} = \frac{1}{2} \longrightarrow a_{n+1} = 2a_n \end{cases}$

$a_1 = 2$

$a_2 = 2a_1 = 2 \cdot 2 = 2^2$

$a_3 = 2a_2 = 2^3$

$\vdots$

$a_n = 2^n$

So, $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} 2^n \leftarrow$ geometric w/ $r = 2$

$\Rightarrow$ diverges

#95 $\begin{cases} a_1 = 10 \\ \frac{a_n}{a_{n+1}} = -10 \longrightarrow a_{n+1} = -\frac{1}{10} a_n \end{cases}$

mult. top+bot by 10^3

$\frac{1}{10^{n-2}} = \frac{10^3}{10^{n-1}}$

$a_1 = 10$

$a_2 = -\frac{1}{10} a_1 = (-1) \cdot 10^0$

$a_3 = -\frac{1}{10} a_2 = (-1)^2 \cdot 10^{-1}$

$a_4 = -\frac{1}{10} a_3 = (-1)^3 \cdot 10^{-2}$

$a_5 = -\frac{1}{10} a_4 = (-1)^4 \cdot 10^{-3}$

$\vdots$

$a_n = (-1)^{n-1} \cdot 10^{-(n-2)}$

$\longrightarrow \sum a_n = \sum_{n=1}^{\infty} (-1)^{n-1} 10^{-(n-2)}$

$= 10^3 \sum_{n=1}^{\infty} \left(-\frac{1}{10}\right)^{n-1}$

converges $\rightarrow$ (geometric w/ $r = -\frac{1}{10}$)

#98 | Goal: write $\frac{\sqrt{x}}{1-x^{3/2}}$ in terms of a geometric series of $\sqrt{x}$

Soln: From $\frac{1}{1-y} = \sum_{n=0}^{\infty} y^n$

$\downarrow y = x^{3/2}$

$$\frac{1}{1-x^{3/2}} = \sum_{n=0}^{\infty} \left(x^{3/2}\right)^n$$

write as $(\sqrt{x})^{3n}$

$\downarrow$ multiply by $\sqrt{x}$

$$\boxed{\frac{\sqrt{x}}{1-x^{3/2}} = \sum_{n=0}^{\infty} (\sqrt{x})^{3n+1}}$$

#102 | $\sum_{n=1}^{\infty} \frac{1}{n^{13}} - \frac{1}{(n+1)^{13}}$

Partial sum:

$$S_k = \left(1 - \cancel{\frac{1}{2^{13}}}\right) + \left(\cancel{\frac{1}{2^{13}}} - \cancel{\frac{1}{3^{13}}}\right) + \left(\cancel{\frac{1}{3^{13}}} - \frac{1}{4^{13}}\right) + \dots + \left(\frac{1}{(k-1)^{13}} - \cancel{\frac{1}{k^{13}}}\right) + \left(\cancel{\frac{1}{k^{13}}} - \frac{1}{(k+1)^{13}}\right)$$

$$= 1 - \frac{1}{(k+1)^{13}}$$

So, $$\sum_{n=1}^{\infty} \frac{1}{n^{13}} - \frac{1}{(n+1)^{13}} = \lim_{k\to\infty} S_k = \lim_{k\to\infty} 1 - \frac{1}{(k+1)^{13}} = 1 - 0 = 1$$

#104 | $\sum_{n=1}^{\infty} \left(\sin(n) - \sin(n+1)\right)$

$$S_k = \left(\sin(1) - \cancel{\sin(2)}\right) + \left(\cancel{\sin(2)} - \cancel{\sin(3)}\right) + \dots + \left(\sin(k-1) - \cancel{\sin(k)}\right) + \left(\cancel{\sin(k)} - \sin(k+1)\right)$$

$$= \sin(1) - \sin(k+1)$$

So, $$\sum_{n=1}^{\infty} \sin(n) - \sin(n+1) = \lim_{k\to\infty} S_k = \lim_{k\to\infty} \sin(1) - \sin(k+1)$$

Limit of this DNE — OSCILLATES

This sum diverges.