

HW8 MATH 2502 Spring 2019

#5.1 #7 Given:  $\begin{cases} a_1 = -3 \\ a_{n-1} - a_n = 4 \text{ for } n \geq 1 \end{cases}$

Soln: By recurrence relation,

$$a_n = a_{n-1} - 4$$

So,

given  $\rightarrow a_1 = -3$

$n=2 \rightarrow a_2 = a_1 - 4 = -3 - 4 = -7$

$n=3 \rightarrow a_3 = a_2 - 4 = -7 - 4 = -11 = -3 - 2(4)$

$n=4 \rightarrow a_4 = a_3 - 4 = -11 - 4 = -15 = -3 - 3(4)$

$n=5 \rightarrow a_5 = a_4 - 4 = -15 - 4 = -19 = -3 - 4(4)$

$n=6 \rightarrow a_6 = a_5 - 4 = -19 - 4 = -23 = -3 - 5(4)$

$n=7 \rightarrow a_7 = a_6 - 4 = -23 - 4 = -27 = -3 - 6(4)$

in general:

$$a_n = -3 - 4(n-1)$$

#8 Given:  $\begin{cases} \frac{a_{n+1}}{a_n} = 10 \\ a_1 = 1 \end{cases}$

Soln: Recurrence relation says  $a_{n+1} = 10a_n$

given  $\rightarrow a_1 = 1$

$n=1 \rightarrow a_2 = 10a_1 = 10(1)$

$n=2 \rightarrow a_3 = 10a_2 = 10^2$

$n=3 \rightarrow a_4 = 10a_3 = 10^3$

$\vdots$   
 $a_n = 10^{n-1}$

14  
17  
23  
26  
27  
28

5.2  
67  
70  
72  
73

#17 
$$\begin{cases} a_1 = 2 \\ a_{n+1} = \frac{(n+1)a_n}{2} \end{cases}$$

$\Rightarrow a_1 = 2$

(n=1)  $\rightarrow a_2 = \frac{2 \cdot a_1}{2} = \frac{2 \cdot 2}{2}$

(n=2)  $\rightarrow a_3 = \frac{3 \cdot a_2}{2} = \frac{3 \cdot (2 \cdot 2)}{2 \cdot 2} = \frac{3 \cdot 2}{2^2} \cdot 2 = \frac{3!}{2}$

(n=3)  $\rightarrow a_4 = \frac{4 \cdot a_3}{2} = \frac{4}{2} \cdot \left( \frac{3 \cdot 2 \cdot 2}{2 \cdot 2} \right) = \frac{4 \cdot 3 \cdot 2}{2^3} \cdot 2 = \frac{4!}{2^2}$

(n=4)  $a_5 = \frac{5 \cdot a_4}{2} = \frac{5}{2} \cdot \frac{4!}{2^2} = \frac{5!}{2^3}$

in general:  $a_n = \frac{(n-1)!}{2^{n-2}}$

#26 if  $\lim a_n = 1$  and  $\lim b_n = -1$ , then

$$\lim_{n \rightarrow \infty} \frac{a_n - b_n}{a_n + b_n} = \frac{1 - (-1)}{1 + (-1)} \sim \frac{2}{0} \text{ DOES NOT EXIST}$$

#27 
$$\lim_{n \rightarrow \infty} \frac{n^2}{2^n} \stackrel{\infty/\infty}{=} \lim_{n \rightarrow \infty} \frac{2n}{\ln(2) \cdot 2^n} \stackrel{\infty/\infty}{=} \lim_{n \rightarrow \infty} \frac{2}{(\ln(2))^2 \cdot 2^n} = 0$$

$$\frac{d}{dx} z^x = \frac{d}{dx} e^{x \ln(z)} = \ln(z) \cdot z^x$$

#28 
$$\lim_{n \rightarrow \infty} \frac{(n-1)^2}{(n+1)^2} \stackrel{0/0}{=} \lim_{n \rightarrow \infty} \frac{2(n-1)}{2(n+1)} \stackrel{0/0}{=} \lim_{n \rightarrow \infty} \frac{2}{2} = 1$$

§5.2 | #70

$$\sin(1) + \sin\left(\frac{1}{2}\right) + \sin\left(\frac{1}{3}\right) + \sin\left(\frac{1}{4}\right) + \dots$$

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$$

#73 |  $a_n = \sin\left(\frac{n\pi}{2}\right)$

$$a_1 = \sin\left(\frac{\pi}{2}\right) = 1$$

$$a_2 = \sin\left(\frac{2\pi}{2}\right) = \sin(\pi) = 0$$

$$a_3 = \sin\left(\frac{3\pi}{2}\right) = -1$$

$$a_4 = \sin\left(\frac{4\pi}{2}\right) = \sin(2\pi) = 0$$

Partial sums

$$S_1 = a_1 = 1$$

$$S_2 = a_1 + a_2 = 1 + 0 = 1$$

$$S_3 = a_1 + a_2 + a_3 = 1 + 0 + (-1) = 0$$

$$S_4 = a_1 + a_2 + a_3 + a_4 = 1 + 0 + (-1) + 0 = 0$$

$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

$\sum_{n=1}^{\infty} \frac{1}{n^3} = \frac{\zeta(3)}{1}$

$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

$\sum_{n=1}^{\infty} \frac{1}{n^5} = \frac{\zeta(5)}{1}$

$\sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945}$

$\sum_{n=1}^{\infty} \frac{1}{n^7} = \frac{\zeta(7)}{1}$

$\sum_{n=1}^{\infty} \frac{1}{n^8} = \frac{\pi^8}{7875}$

$\sum_{n=1}^{\infty} \frac{1}{n^9} = \frac{\zeta(9)}{1}$

$\sum_{n=1}^{\infty} \frac{1}{n^{10}} = \frac{\pi^{10}}{93456}$

$\sum_{n=1}^{\infty} \frac{1}{n^{11}} = \frac{\zeta(11)}{1}$

$\sum_{n=1}^{\infty} \frac{1}{n^{12}} = \frac{\pi^{12}}{620100}$

$\sum_{n=1}^{\infty} \frac{1}{n^{13}} = \frac{\zeta(13)}{1}$

$\sum_{n=1}^{\infty} \frac{1}{n^{14}} = \frac{\pi^{14}}{1352880}$

$\sum_{n=1}^{\infty} \frac{1}{n^{15}} = \frac{\zeta(15)}{1}$

$\sum_{n=1}^{\infty} \frac{1}{n^{16}} = \frac{\pi^{16}}{18253200}$

$\sum_{n=1}^{\infty} \frac{1}{n^{17}} = \frac{\zeta(17)}{1}$

$\sum_{n=1}^{\infty} \frac{1}{n^{18}} = \frac{\pi^{18}}{313849200}$

$\sum_{n=1}^{\infty} \frac{1}{n^{19}} = \frac{\zeta(19)}{1}$

$\sum_{n=1}^{\infty} \frac{1}{n^{20}} = \frac{\pi^{20}}{6355132800}$