HW8 MATH 2502 Spring 2019

§5.1 #7 Given: $\begin{cases} a_1 = -3 \\ a_{n-1} - a_n = 4 & \text{for } n \geq 1 \end{cases}$

Soln: By recurrence relation,

$$a_n = a_{n-1} - 4$$

So,

given $\longrightarrow a_1 = -3$

$n=2 \longrightarrow a_2 = a_1 - 4 = -3-4 = -7$

$n=3 \longrightarrow a_3 = a_2 - 4 = -7-4 = -11 = -3-2(4)$

$n=4 \longrightarrow a_4 = a_3 - 4 = -11-4 = -15 = -3-3(4)$

$n=5 \longrightarrow a_5 = a_4 - 4 = -15-4 = -19 = -3-4(4)$

$n=6 \longrightarrow a_6 = a_5 - 4 = -19-4 = -23 = -3-5(4)$

$n=7 \longrightarrow a_7 = a_6 - 4 = -23-4 = -27 = -3-6(4)$

in general: $\boxed{a_n = -3 - 4(n-1)}$

#8 Given: $\begin{cases} \frac{a_{n+1}}{a_n} = 10 \\ a_1 = 1 \end{cases}$

Soln: Recurrence relation says $a_{n+1} = 10a_n$

given $\longrightarrow a_1 = 1$

$n=1 \longrightarrow a_2 = 10a_1 = 10(1)$

$n=2 \longrightarrow a_3 = 10a_2 = 10^2$

$n=3 \longrightarrow a_4 = 10a_3 = 10^3$

$\vdots$

$\boxed{a_n = 10^{n-1}}$

14 5.2
17 67
23 70
26 72
27 73
28

#17 $\begin{cases} a_1 = 2 \\ a_{n+1} = \frac{(n+1)a_n}{2} \end{cases}$

$\Rightarrow a_1 = 2$

$(n=1) \rightarrow a_2 = \frac{2 \cdot a_1}{2} = \frac{2 \cdot 2}{2}$

$(n=2) \rightarrow a_3 = \frac{3 \cdot a_2}{2} = \frac{3}{2} \cdot \left(\frac{2 \cdot 2}{2}\right) = \frac{3 \cdot 2}{2^2} \cdot 2 = \frac{3!}{2}$

$(n=3) \rightarrow a_4 = \frac{4 \cdot a_3}{2} = \frac{4}{2} \cdot \left(\frac{3}{2} \cdot \frac{2 \cdot 2}{2}\right) = \frac{4 \cdot 3 \cdot 2}{2^3} \cdot 2 = \frac{4!}{2^2}$

$(n=4)$ $a_5 = \frac{5 \cdot a_4}{2} = \frac{5}{2} \cdot \frac{4!}{2^2} = \frac{5!}{2^3}$

in general: $a_n = \frac{(n-1)!}{2^{n-2}}$

#26 if $\lim a_n = 1$ and $\lim b_n = -1$, then

$$\lim_{n\to\infty} \frac{a_n - b_n}{a_n + b_n} = \frac{1-(-1)}{1+(-1)} \sim \frac{2}{0} \text{ DOES NOT EXIST}$$

#27 $$\lim_{n\to\infty} \frac{n^2}{2^n} \overset{\frac{\infty}{\infty}}{\underset{L.H.}{=}} \lim_{n\to\infty} \frac{2n}{\ln(2)\cdot 2^n} \overset{\frac{\infty}{\infty}}{\underset{L.H.}{=}} \lim_{n\to\infty} \frac{2}{(\ln(2))^2 \cdot 2^n} = 0$$

$\frac{d}{dx} 2^x = \frac{d}{dx} e^{x\ln(2)}$
$= \ln(2) \cdot 2^x$

#28 $$\lim_{n\to\infty} \frac{(n-1)^2}{(n+1)^2} \overset{L'H}{=} \lim_{n\to\infty} \frac{2(n-1)}{2(n+1)} \overset{L.H.}{=} \lim_{n\to\infty} \frac{2}{2} = 1$$

$\frac{\infty}{\infty}$ $\frac{\infty}{\infty}$

§5.2 #70

$\sin(1)+\sin(\frac{1}{2})+\sin(\frac{1}{3})+\sin(\frac{1}{4})+\cdots$

$$= \sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$$

#73 $a_n = \sin\left(\frac{n\pi}{2}\right)$

$a_1 = \sin(\frac{\pi}{2}) = 1$

$a_2 = \sin(\frac{2\pi}{2}) = \sin(\pi) = 0$

$a_3 = \sin(\frac{3\pi}{2}) = -1$

$a_4 = \sin(\frac{4\pi}{2}) = \sin(2\pi) = 0$

Partial sums

$S_1 = a_1 = 1 = 1$

$S_2 = a_1 + a_2 = 1+0 = 1$

$S_3 = a_1 + a_2 + a_3 = 1+0+(-1) = 0$

$S_4 = a_1 + a_2 + a_3 + a_4 = 1+0+(-1)+0 = 0$