HW6 MATH 2502 Spring 2019

§3.3 #134 $\int \frac{1}{\sqrt{4-x^2}}\,dx = \int \frac{2\cos(\theta)}{\sqrt{4-4\sin^2(\theta)}}\,d\theta$

$a=2$

$x = 2\sin(\theta)$

$dx = 2\cos(\theta)$

$= \frac{2}{2}\int \frac{\cos\theta}{\cos\theta}\,d\theta$

$= \int 1\,d\theta$

$= \theta + C$

$\frac{x}{2} = \sin(\theta)$

$\theta = \arcsin\left(\frac{x}{2}\right)$

$= \arcsin\left(\frac{x}{2}\right) + C$

#136 $\int \sqrt{4-x^2}\,dx = \int \sqrt{4-4\sin^2\theta}\,(2\cos(\theta))\,d\theta$ ($\sqrt{4-4\sin^2\theta} = 2\cos(\theta)$)

$x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

$= 4\int \cos^2(\theta)\,d\theta$

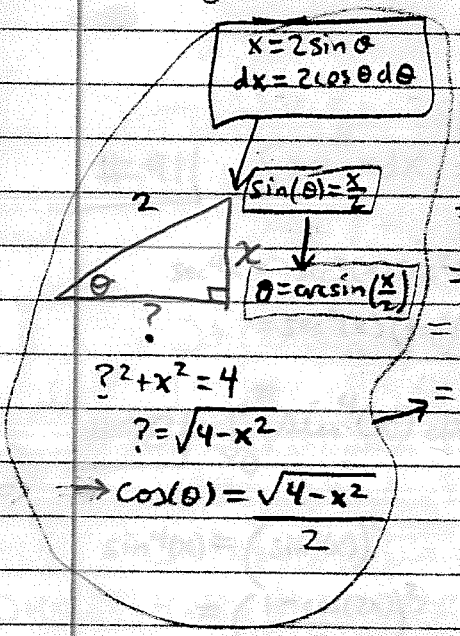
$= 4\int \frac{1-\cos(2\theta)}{2}\,d\theta$

$= 2\int 1\,d\theta - 2\int \cos(2\theta)\,d\theta$

$= 2\theta - \sin(2\theta) + C$

$= 2\theta - 2\sin(\theta)\cos(\theta) + C$

$= 2\arcsin\left(\frac{x}{2}\right) - 2\left(\frac{x}{2}\right)\left(\frac{\sqrt{4-x^2}}{2}\right) + C$



$?^2 + x^2 = 4$

$? = \sqrt{4-x^2}$

$\rightarrow \cos(\theta) = \frac{\sqrt{4-x^2}}{2}$

Recall

$\frac{d}{dx}\cot(x) = \frac{d}{dx}\frac{\cos x}{\sin x} = \frac{-\sin^2(x)-\cos^2(x)}{\sin^2(x)} = -\csc^2(x)$

S + C

#139 $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx = \int \frac{\cos(\theta)}{\sin^2(\theta)\cos(\theta)}\,d\theta$

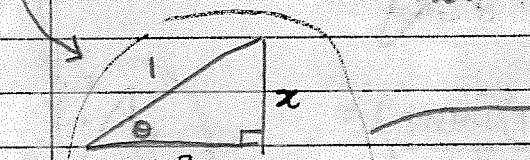
$x = \sin\theta$

$dx = \cos\theta\,d\theta \rightarrow \sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \cos(\theta)$

$= \int \csc^2(\theta)\,d\theta$

$= -\cot(\theta) + C$

$= -\frac{\sqrt{1-x^2}}{x} + C$



$?^2 + x^2 = 1$

$? = \sqrt{1-x^2}$

$\Rightarrow \cos(\theta) = \sqrt{1-x^2}$

$\Rightarrow \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{\sqrt{1-x^2}}{x}$

$s^2 + c^2 = 1$

$1 + t^2 = \sec^2$

#142 $\int \frac{\sqrt{x^2-25}}{x}\,dx = \int \frac{\sqrt{25\sec^2(\theta)-25}}{5\sec(\theta)}\,d\theta$

$\sqrt{25\sec^2(\theta)-25} = 5\sqrt{\sec^2(\theta)-1} = 5\tan(\theta)$

$x = 5\sec(\theta)$

$dx = 5\sec(\theta)\tan(\theta)\,d\theta$

$= \frac{5}{5}\int \frac{\tan(\theta)}{\sec(\theta)}\,d\theta$

hyp. → x, adj. → 5: $\frac{x}{5} = \sec(\theta)$

$= \int \frac{\frac{\sin\theta}{\cos(\theta)}}{\cos\theta}\,d\theta$

$\frac{5}{x} = \frac{1}{\sec(\theta)} = \frac{1}{\frac{1}{\cos(\theta)}} = \cos(\theta)$

$= \int \sin\theta\,d\theta = -\cos(\theta) + C$

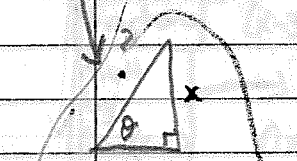
$= -\frac{5}{x} + C$

#148 $\int \frac{\sqrt{1+x^2}}{x}\,dx = \int \frac{\sqrt{1+\tan^2\theta}}{\tan(\theta)}\sec^2(\theta)\,d\theta$

$= \sqrt{\sec^2\theta} = \sec(\theta)$

$x = \tan\theta$

$dx = \sec^2(\theta)\,d\theta$



$1^2 + x^2 = ?^2$

$\sqrt{1+x^2} = ?$

$\Downarrow$

$\csc(\theta) = \frac{\sqrt{1+x^2}}{x}$

$\cot(\theta) = \frac{1}{x}$

$\sec(\theta) = \sqrt{1+x^2}$

$= \int \frac{\sec(\theta)}{\tan(\theta)}\sec^2(\theta)\,d\theta$

$1+\tan^2(\theta)$

$= \int \frac{\sec(\theta)\left(1+\tan^2(\theta)\right)}{\tan(\theta)}\,d\theta$

$= \int \frac{\sec(\theta)}{\tan(\theta)}\,d\theta + \int \frac{\sec(\theta)\tan^2(\theta)}{\tan(\theta)}\,d\theta$

$= \int \frac{\frac{1}{\cos(\theta)}}{\left(\frac{\sin(\theta)}{\cos(\theta)}\right)}\,d\theta + \int \sec(\theta)\tan(\theta)\,d\theta$

$= \int \csc(\theta)\,d\theta + \int \frac{d}{d\theta}\sec(\theta)\,d\theta$

$= \ln\left(\csc(\theta) - \cot(\theta)\right) + \sec(\theta) + C$

$= \ln\left(\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right) + \sqrt{1+x^2} + C$

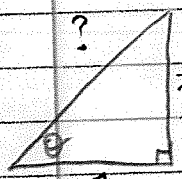
#151 $\int \frac{1}{x^2\sqrt{x^2+1}}\,dx = \int \frac{\sec^2(\theta)}{\tan^2(\theta)\sqrt{\tan^2(\theta)+1}}\,d\theta$

$x = \tan\theta$

$dx = \sec^2(\theta)\,d\theta$

$= \int \frac{\sec^2(\theta)}{\tan^2(\theta)\sec(\theta)}\,d\theta$

$= \int \frac{\frac{1}{\cos(\theta)}}{\frac{\sin^2(\theta)}{\cos^2(\theta)}}\,d\theta$



$= \int \frac{\cos^2(\theta)}{\cos(\theta)} \cdot \frac{1}{\sin^2(\theta)}\,d\theta$

$= \int \cot(\theta)\csc(\theta)\,d\theta$

$1^2 + x^2 = ?^2$

$? = \sqrt{1+x^2}$

$= -\csc(\theta) + C$

$\csc(\theta) = \frac{\sqrt{1+x^2}}{x}$

$= -\frac{\sqrt{1+x^2}}{x} + C$

recall

$\frac{d}{dx}\csc(x) = \frac{d}{dx}\frac{1}{\sin x}$

$= -\frac{\cos x}{\sin^2 x}$

$= -\cot(x)\csc(x)$