don't use v — already in use

HW5 MATH 2502 Spring 2019

§3.1 #6

$$\int v\sin(v)\,dv \qquad \begin{array}{ll} u=v, & dw=\sin(v)\,dv \\ du=dv & w=-\cos(v) \end{array}$$

$$= -v\cos(v) + \int \cos(v)\,dv$$

$$= -v\cos(v) + \sin(v) + C$$

#10

$$\int x^2e^x\,dx = x^2e^x - 2\int xe^x\,dx \qquad (\star)$$

$u=x^2$ $dv=e^x dx$

$du=2x dx$ $v=e^x$

$$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C$$

$u_2=x$ $dv_2=e^x dx$

$du_2=dx$ $v_2=e^x$

Therefore, by $(\star)$,

$$\boxed{\int x^2e^x\,dx = x^2e^x - 2[xe^x] + 2e^x + C}$$

#12

$$\int xe^{4x}\,dx = \frac{x}{4}e^{4x} - \int \frac{1}{4}e^{4x}\,dx$$

$$= \frac{x}{4}e^{4x} - \frac{1}{16}e^{4x} + C$$

$u=x$ $dv=e^{4x}dx$

$du=dx$ $v=\frac{1}{4}e^{4x}$

#20

$u=e^x \to du=e^x dx$

$dv=\cos x\,dx \to v=\sin x$

$u_2=e^x \to du_2=e^x dx$

$dv_2=\sin x\,dx \to v_2=-\cos x$

$$\int e^x\cos x\,dx = e^x\sin(x) - \int e^x\sin(x)\,dx$$

$$= e^x\sin(x) - \left[-e^x\cos(x) + \int e^x\cos x\,dx\right]$$

$$\Rightarrow \int e^x\cos x\,dx = e^x\sin(x) + e^x\cos(x) - \int e^x\cos x\,dx$$

$$\Rightarrow 2\int e^x\cos x\,dx = e^x(\sin(x)+\cos(x))$$

$$\Rightarrow \int e^x\cos x\,dx = \frac{e^x(\sin(x)+\cos x)}{2}$$

no int by parts needed

$u=-x^2 \to -\frac{1}{2}du = x\,dx$

#21 $\int x e^{-x^2}\,dx = -\frac{1}{2}\int e^u\,du$

$= -\frac{1}{2}e^u + C$

$= -\frac{1}{2}e^{-x^2} + C$

$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

$\ln\left(\frac{1}{e}\right) = \underbrace{\ln(1)}_{=0} - \underbrace{\ln(e)}_{=1}$

$= -1$

#38 $\int_{1/e}^{1} \ln(x)\,dx = x\ln(x)\Big|_{1/e}^{1} - \int_{1/e}^{1} x\cdot\frac{1}{x}\,dx$

$u = \ln x \quad dv = 1\,dx$
$du = \frac{1}{x}dx \quad v = x$

$= \left(1(\underbrace{\ln(1)}_{=0}) - \frac{1}{e}\ln\left(\frac{1}{e}\right)\right) - \int_{1/e}^{1} 1\,dx$

$= -\frac{1}{e}(-1) - x\Big|_{1/e}^{1}$

$= \frac{1}{e} - \left(1 - \frac{1}{e}\right)$

$= \frac{2}{e} - 1$

§3.2 #80 $\int \cos^3 x\,dx = \int \cos(x)(1-\sin^2 x)\,dx$

$= \int \cos x\,dx - \int \cos(x)\sin^2(x)\,dx$

$= \sin(x) + \int u\,du$

$= \sin(x) + \frac{u^2}{2} + C$

$= \sin(x) + \frac{1}{2}\cos^2(x) + C$

$u = \cos x$
$-du = \sin x\,dx$

$\boxed{\cos^3 x = \cos(x)\cdot\cos^2(x) = \cos(x)(1-\sin^2 x)}$

#82 $\int \cos^5(x)\,dx = \int \cos(x)\left(1 - 2\sin^2(x) + \sin^4(x)\right)dx$

$= \int 1 - 2u^2 + u^4\,du$

$u = \sin x$
$du = \cos x\,dx$

$= u - \frac{2}{3}u^3 + \frac{1}{5}u^5 + C$

$= \sin(x) - \frac{2}{3}\sin^3(x) + \frac{1}{5}\sin^5(x) + C$

$\cos^5 x = \cos(x)\left(\cos^2(x)\right)^2$

$= \cos(x)\left(1-\sin^2(x)\right)^2$

$= \cos(x)\left(1 - 2\sin^2(x) + \sin^4(x)\right)$

#84 $\int \sin^3 x \cos^3 x \, dx = \int \sin(x)(1-\cos^2(x))\cos^3(x)\,dx$

$= \int \sin(x)[\cos^3 x - \cos^5 x]\,dx$

$\sin^3(x) = \sin(x)\sin^2(x)$

$= \sin(x)(1-\cos^2(x))$

$u = \cos x$

$-du = \sin x \, dx$

$= -\int u^3 - u^5 \, du$

$= -\frac{u^4}{4} + \frac{u^6}{6} + C$

$= -\frac{\cos^4(x)}{4} + \frac{\cos^6(x)}{6} + C$

#90 $\int \tan(x)\sec^3(x)\,dx = \int \sec^2(x)\sec(x)\tan(x)\,dx$

$u = \sec x$

$du = \sec(x)\tan(x)\,dx$

$\tan(x)\sec^3(x) = \tan(x)\sec(x)\sec^2(x)$

$= \int u^2 \, du$

$= \frac{u^3}{3} + C$

$= \frac{\sec^3 x}{3} + C$

$s^2 + c^2 = 1$

$t^2 + 1 = \sec^2$

#91 $\int \sec^4 x \, dx = \int \sec^2(x)(1+\tan^2(x))\,dx$

$= \int 1 + u^2 \, du = u + \frac{u^3}{3} + C$

$\sec^4 x = \sec^2(x)\sec^2 x$

$= \sec^2(x)(\tan^2(x)+1)$

$u = \tan x$

$du = \sec^2 x \, dx$

$= \tan(x) + \frac{1}{3}\tan^3(x) + C$

#98 $\int_0^{\pi} \sin^4(x)\,dx = \frac{1}{4}\int_0^{\pi} \frac{3}{2} - 2\cos(2x) + \frac{1}{4}\cos(4x)\,dx$

$\sin^4(x) = (\sin^2(x))^2$

$= \left(\frac{1-\cos(2x)}{2}\right)^2$

$= \frac{1}{4}\left[\frac{3}{2}x - \sin(2x) + \frac{1}{16}\sin(4x)\right]_0^{\pi}$

$= \frac{1}{4}\left[\left(\frac{3}{2}\pi - \underbrace{\sin(2\pi)}_{=0} + \frac{1}{16}\underbrace{\sin(4\pi)}_{=0}\right) - 0\right]$

$= \frac{3\pi}{8}$

$= \frac{1}{4}(1 - 2\cos(2x) + \cos^2(2x))$

$= \frac{1}{4}\left(1 - 2\cos(2x) + \frac{1+\cos(4x)}{2}\right)$

$= \frac{1}{4}\left(\frac{3}{2} - 2\cos(2x) + \frac{1}{2}\cos(4x)\right)$

identity on p. 278

(cosine "even")

#104 $\displaystyle\int_0^{\pi} \sin(3x)\sin(5x)\,dx = \int_0^{\pi} \frac{1}{2}\Big[\underbrace{\cos((3-5)x)}_{=\cos(-2x)=\cos(2x)} - \cos((3+5)x)\Big]\,dx$

$$= \frac{1}{2}\int_0^{\pi} \cos(2x) - \cos(8x)\,dx$$

$$= \frac{1}{2}\left[\frac{1}{2}\sin(2x) - \frac{1}{8}\sin(8x)\right|_0^{\pi}$$

$$= \frac{1}{2}\left[\left(\frac{1}{2}\cancel{\sin(2\pi)}^{0} - \frac{1}{8}\cancel{\sin(8\pi)}^{0}\right) - 0\right]$$

$$= 0$$