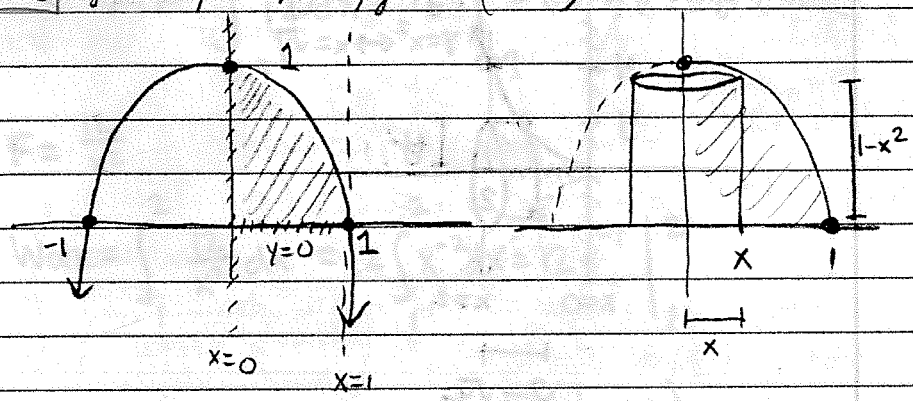
§2.3: 120, 122, 124, 125, 131, 132

§2.5: 218, 222, 224, 226, 229, 231

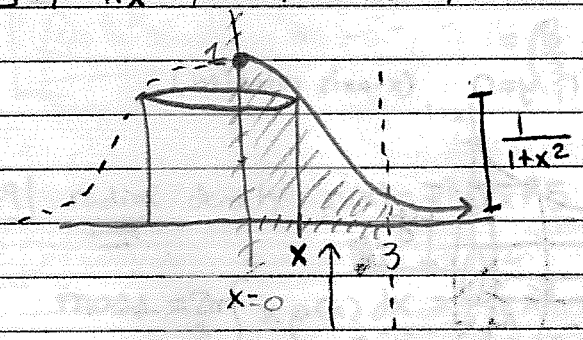
HW4 MATH 2502 Spring 2019

§2.3 #120 $y = 1-x^2$, $x=0$, $x=1$, $y=0$ $(x>0)$ (y-axis rotation)



$$Vol = 2\pi \int_0^1 x(1-x^2)\,dx = 2\pi \int_0^1 x - x^3\,dx$$
$$= 2\pi \left[\frac{x^2}{2} - \frac{x^4}{4}\Big|_0^1\right]$$
$$= 2\pi \left[\left(\frac{1}{2} - \frac{1}{4}\right) - 0\right]$$
$$= \frac{2\pi}{4} = \frac{\pi}{2}$$

#124 $y = \frac{1}{1+x^2}$, $x=0$, $x=3$, $y=0$, $(x>0)$ (y-axis rotation)



$x=0 \to u = 1+0^2 = 1$

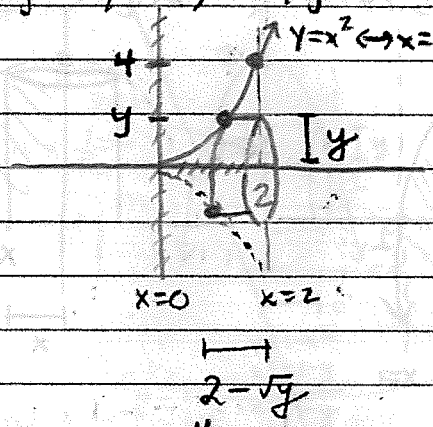
$x=3 \to u = 1+3^2 = 10$

$u = 1+x^2$

$\frac{1}{2}du = x\,dx$

$$Vol = 2\pi \int_0^3 \frac{x}{1+x^2}\,dx = \frac{2\pi}{2}\int_1^{10} \frac{1}{u}\,du$$
$$= \pi \ln(u)\Big|_1^{10}$$
$$= \pi \ln(10) - \pi \ln(1) \quad (\ln(1) = 0)$$
$$= \pi \ln(10)$$

#131 $y=x^2, x=0, x=2, y=0$ (x-axis rotation)

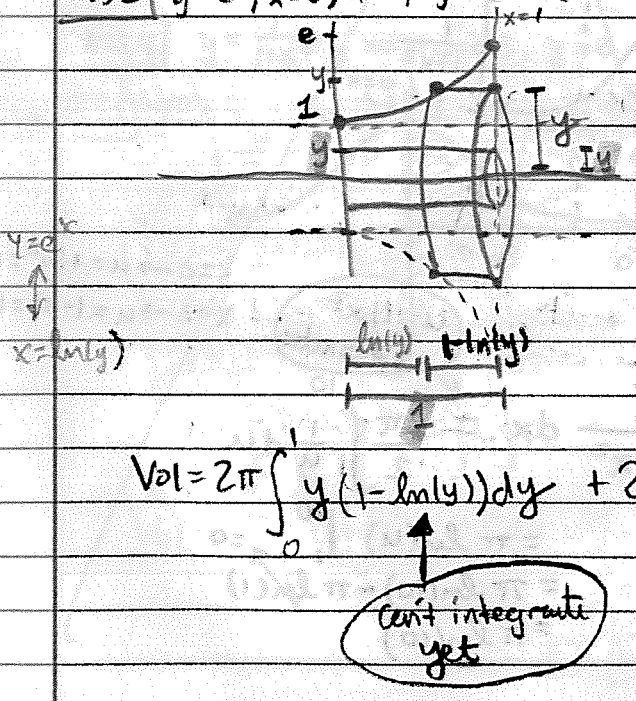


$$Vol = 2\pi \int_0^4 y(2-\sqrt{y})\,dy = 2\pi \int_0^4 2y - y^{3/2}\,dy$$

$$= 2\pi \left[y^2 - \frac{y^{5/2}}{5/2} \right]_0^4$$

$$= 2\pi \left[\left(16 - \frac{2}{5}\cdot 4^{5/2}\right) - 0 \right]$$

#132 $y=e^x, x=0, x=1, y=0$ (x-axis rotation)



$$Vol = 2\pi \int_0^1 y(1-\ln(y))\,dy + 2\pi \int_1^e y(1)\,dy$$

can't integrate yet

§2.5 #218 $F = 12$

$$\text{Work} = \int_{0.9}^{1.1} 12\,dx = 12x\Big|_{0.9}^{1.1} = 12(1.1) - 12(0.9)$$

#222 $F = \frac{12}{x^2}$

$$\text{Work} = \int_1^2 \frac{12}{x^2}\,dx = 12\int_1^2 x^{-2}\,dx = 12\frac{x^{-1}}{-1}\Big|_1^2$$

$$= -12\left(\frac{1}{2} - 1\right)$$

$$= 6$$

#224 $\rho(x) = x^2 + 2x \ \frac{lb}{ft}$

$$\text{mass} = \int_0^2 \rho(x)\,dx = \int_0^2 x^2 + 2x\,dx = \frac{x^3}{3} + x^2\Big|_0^2$$

$$= \left(\frac{8}{3} + 4\right) - 0$$

#229 radial density $\rho(x) = x^3 - 2x + 5$

$$\text{mass} = 2\pi\int_0^2 x\rho(x)\,dx = 2\pi\int_0^2 x^4 - 2x^2 + 5x\,dx$$

because radial density, not linear density

$$= 2\pi\left[\frac{x^5}{5} - \frac{2}{3}x^3 + \frac{5}{2}x^2\Big|_0^2\right.$$

$$= 2\pi\left(\frac{32}{5} - \frac{2}{3}(8) + \frac{5}{2}(4)\right) - 0$$