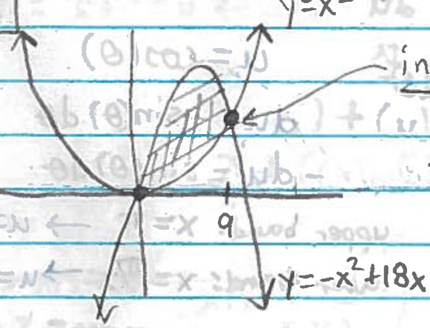
HW3 MATH 2502 Spring 2019

§2.1 #7



intersection pts

$x^2=-x^2+18x$

$2x^2-18x=0$

$x(2x-18)=0$

$x=0$ $\quad$ $2x-18=0$

$x=9$

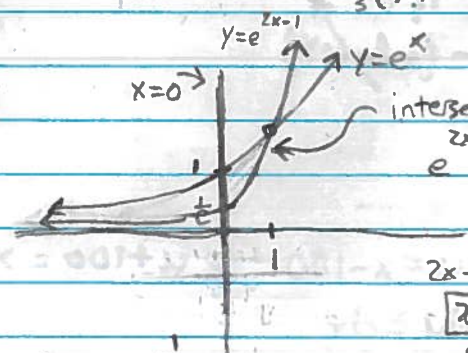
$$\text{Area} = \int_0^9 \text{top} - \text{bottom}\, dx = \int_0^9 (-x^2+18x) - x^2\, dx$$

$$= \int_0^9 -2x^2+18x\, dx$$

$$= -\frac{2}{3}x^3+9x^2 \Big|_0^9$$

$$= -\frac{2}{3}(9^2)+9(9^2)$$

#10



intersection pt

$e^{2x-1}=e^x$

$\downarrow \ln$

$2x-1=x$

$\boxed{x=1}$

$u=2x-1$

$\frac{1}{2}du=dx$

upper bd: $x=0\to u=-1$

lower bd: $x=1\to u=1$

$$\text{Area} = \int_0^1 e^x - e^{2x-1}\, dx = \int_0^1 e^x dx - \int_0^1 e^{2x-1} dx$$

$$= e^x\Big|_0^1 - \frac{1}{2}\int_{-1}^1 e^u du$$

$$= (e-1) - \frac{1}{2}e^u\Big|_{-1}^1$$

$$= (e-1) - \frac{1}{2}(e-e^{-1})$$

intersection

$\sqrt{x} = 12 - x$

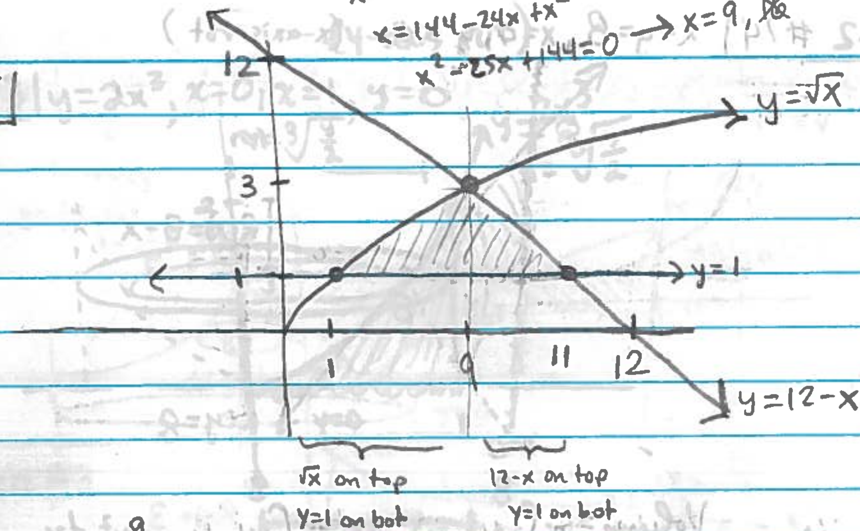
$x = (12-x)^2$

$x = 144 - 24x + x^2$

$x^2 - 25x + 144 = 0 \longrightarrow x = 9, \cancel{16}$

not a soln

#15 $y = \sqrt{x}$, $y = 12 - x$, $y = 1$

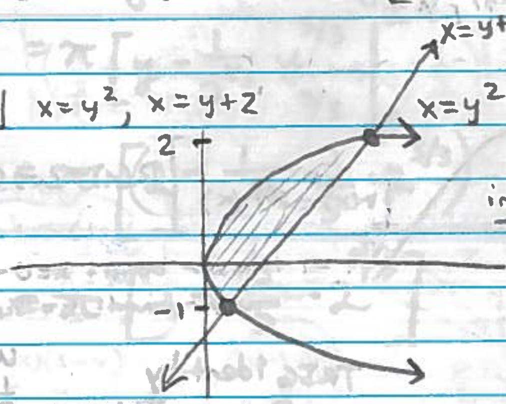


$$\text{Area} = \int_1^9 \sqrt{x} - 1\,dx + \int_9^{12} \underbrace{(12-x)-1}_{11-x}\,dx$$

$$= \frac{x^{3/2}}{3/2} - x \Big|_1^9 + 11x - \frac{x^2}{2}\Big|_9^{12}$$

$$= \left[\left(\frac{2}{3}9^{3/2} - 9\right) - \left(\frac{2}{3} - 1\right)\right] + \left[\left(11(12) - \frac{12^2}{2}\right) - \left(11(9) - \frac{9^2}{2}\right)\right]$$

#23 $x = y^2$, $x = y + 2$



intersection pts

$y^2 = y + 2$

$y^2 - y - 2 = 0$

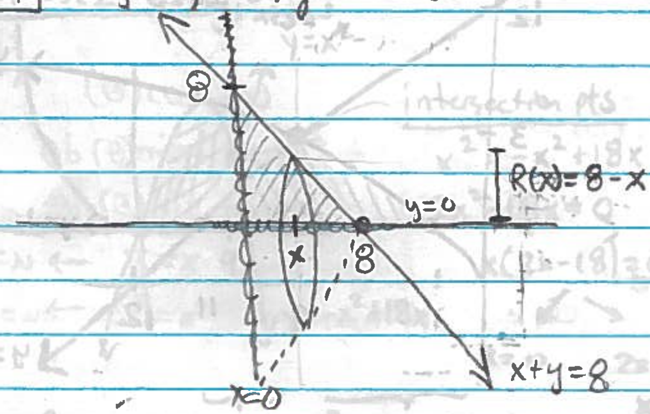
$(y-2)(y+1) = 0$

$y = 2, -1$

$$\text{Area} = \int_{-1}^{2} (y+2) - y^2\,dy = \frac{y^2}{2} + 2y - \frac{y^3}{3}\Big|_{-1}^{2}$$

$$= \left(\frac{2^2}{2} + 2(2) - \frac{2^3}{3}\right) - \left(\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3}\right)$$

§2.2 #74 $x+y=8,\ x=0,\ y=0$ (x-axis rot)

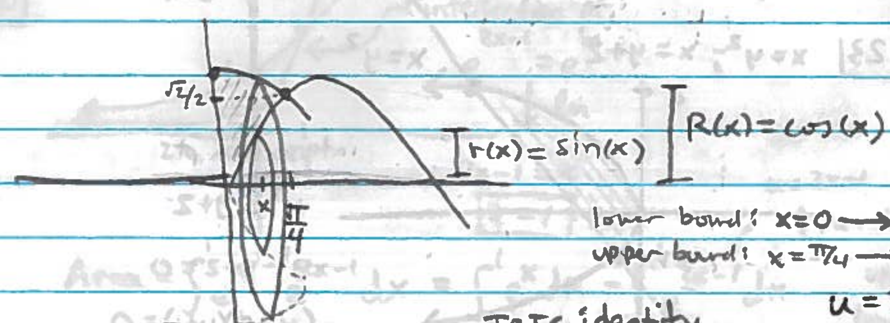


$$\text{Volume} = \pi\int_0^8 (8-x)^2\,dx = \int_0^8 64-16x+x^2\,dx$$

$$= 64x - 8x^2 + \frac{x^3}{3}\Big|_0^8$$

$$= \left(64(8) - 8(8^2) + \frac{8^3}{3}\right) - 0$$

#79 $y=\sin(x),\ y=\cos(x),\ x=0$



lower bound: $x=0 \longrightarrow u=2(0)=0$

upper bound: $x=\pi/4 \longrightarrow u=2(\pi/4)=\frac{\pi}{2}$

$u=2x$

$\frac{1}{2}du=dx$

TRIG identity

$$\text{Vol} = \pi\int_0^{\pi/4} \cos^2(x)-\sin^2(x)\,dx \overset{\text{TRIG identity}}{=} \pi\int_0^{\pi/4}\cos(2x)\,dx$$

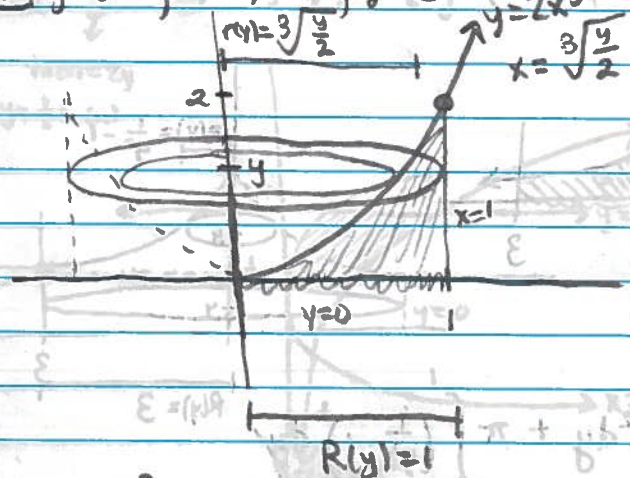
$$= \frac{\pi}{2}\int_0^{\pi/2}\cos(u)\,du$$

$$= \frac{\pi}{2}\left[\sin(u)\Big|_0^{\pi/2}\right] = \frac{\pi}{2}\left[\underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1} - \underbrace{\sin(0)}_{=0}\right]$$

$$= \frac{\pi}{2}$$

(y-axis rot)

#83 $y=2x^3$, $x=0$, $x=1$, $y=0$



$$Vol=\pi\int_0^2 1^2-\left(\sqrt[3]{\frac{y}{2}}\right)^2 dy$$

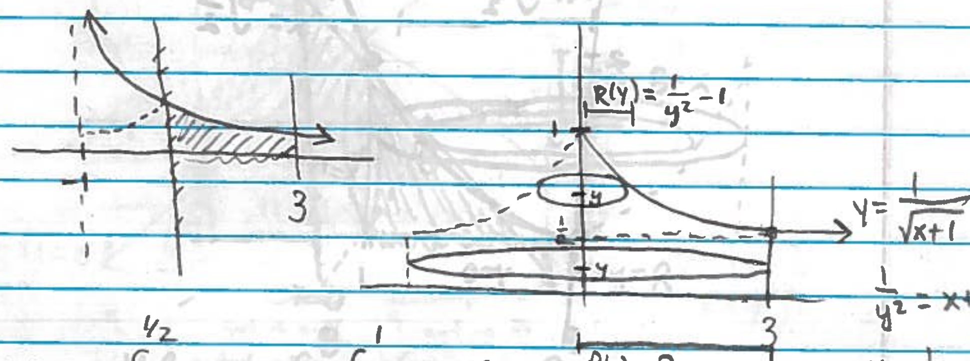
$$=\pi\int_0^2 1-\frac{1}{2^{2/3}}y^{2/3}\,dy$$

$$=\pi\left[y-\frac{1}{2^{2/3}}\frac{y^{5/3}}{5/3}\right]_0^2$$

$$=\pi\left[\left(2-\frac{1}{2^{2/3}}\cdot\frac{3}{5}\cdot 2^{5/3}\right)-0\right]$$

$$=\pi\left[2-\frac{3}{5}\cdot 2^{4/3}\right]$$

#86 $y=\frac{1}{\sqrt{x+1}}$, $x=0$, $x=3$ (y-axis)



$\frac{1}{y^2}=x+1$

$x=\frac{1}{y^2}-1$

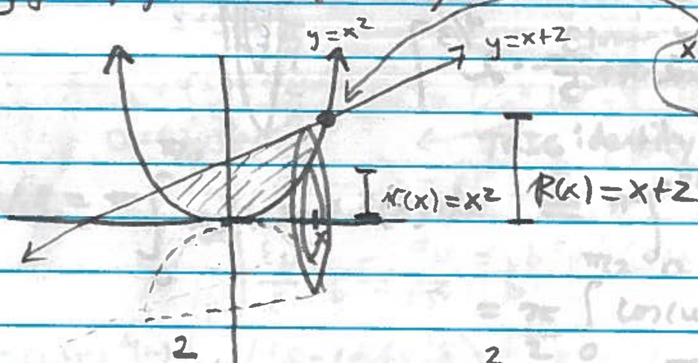
$$Vol=\pi\int_0^{1/2}3^2\,dy+\pi\int_{1/2}^{1}\underbrace{\left(\frac{1}{y^2}-1\right)^2}_{=\frac{1}{y^4}-\frac{2}{y^2}+1=y^{-4}-2y^{-2}+1}dy$$

$$=\pi\int_0^{1/2}9\,dy+\pi\int_{1/2}^{1}y^{-4}-2y^{-2}+1\,dy$$

$$=\pi\left[9y\Big|_0^{1/2}\right.+\pi\left[\frac{y^{-3}}{-3}-\frac{2y^{-1}}{(-1)}+y\Big|_{1/2}^{1}\right.$$

$$=\pi\left[\frac{9}{2}-0\right]+\pi\left[\left(-\frac{1}{3}+2+1\right)-\left(\frac{8}{3}+4+\frac{1}{2}\right)\right]$$

#91 $y=x^2$, $y=x+2$ (x-axis)

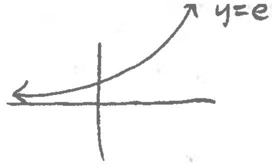


intersection
$x+2=x^2$
$x^2-x-2=0$
$(x-2)(x+1)=0$
$x=2$

$$Vol=\pi\int_0^2\underbrace{(x+2)^2-(x^2)^2}_{=x^2+4x+2-x^4}dx=\pi\int_0^2-x^4+x^2+4x+2\,dx$$

$$=\pi\left[-\frac{x^5}{5}+\frac{x^3}{3}+2x^2+2x\Big|_0^2\right]$$

$$=\pi\left[\left(-\frac{32}{5}+\frac{8}{3}+8+4\right)-0\right]$$

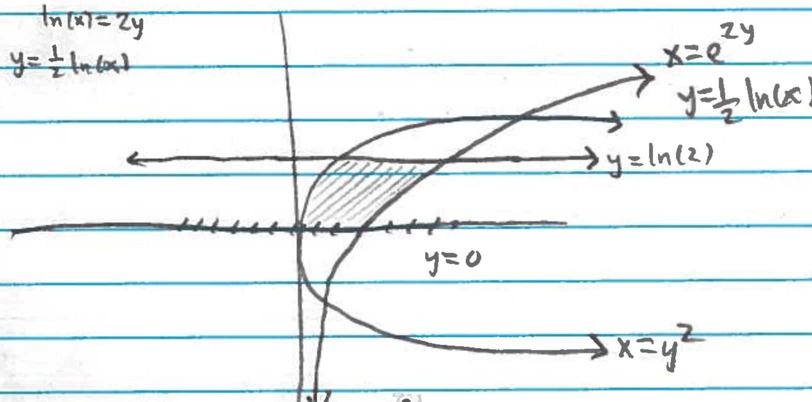


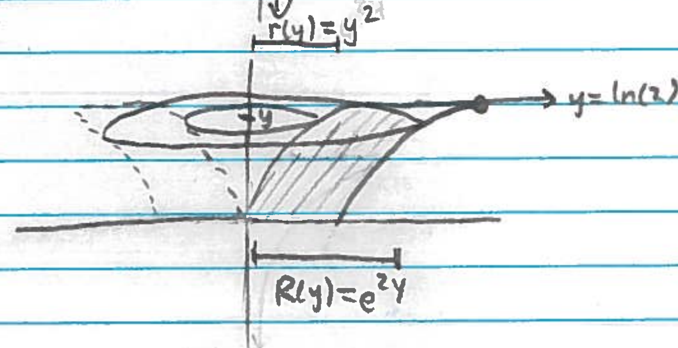
$y=\ln(2) \to x=e^{2\ln(2)} = e^{\ln(4)} = 4$

#101 | $x=e^{2y}$, $x=y^2$, $y=0$, $y=\ln(2)$ (revolve y-axis)

$\ln(x)=2y$

$y=\frac{1}{2}\ln(x)$





$$\text{Vol} = \pi\int_0^{\ln(2)} (e^{2y})^2 - (y^2)^2\,dy$$

$4\ln(2) = \ln(2^4)$

$$= \pi\int_0^{\ln(2)} e^{4y} - y^4\,dy = \pi\left[\frac{1}{4}e^{4y} - \frac{y^5}{5}\right]_0^{\ln(2)}$$

u-sub
u=4y

$$= \pi\left[\left(\frac{1}{4}e^{4\ln(2)} - \frac{(\ln(2))^5}{5}\right) - 0\right]$$

$$= \pi\left[\frac{1}{4}e^{\ln(16)} - \left(\frac{\ln(2)}{5}\right)^5\right]$$

$$= \pi\left[4 - \left(\frac{\ln(2)}{5}\right)^5\right]$$