HW2 MATH 2502 Spring 2019

§1.3 #148) $\frac{d}{dx}\int_1^x e^{-t^2}dt = e^{-x^2}$

chain rule

#154) $\frac{d}{dx}\int_1^{\sin x}\sqrt{1-t^2}\,dt = \left(\sqrt{1-\sin^2(x)}\right)(\cos(x))$

#170) $\int_{-1}^{2} x^2-3x\,dx = \frac{x^3}{3}-\frac{3x^2}{2}\Big|_{-1}^{2} = \left(\frac{2^3}{3}-\frac{3(2^2)}{2}\right)-\left(\frac{(-1)^3}{3}-\frac{3(-1)^2}{2}\right)$

$= \left(\frac{8}{3}-6\right)-\left(-\frac{1}{3}-\frac{3}{2}\right)$

$= \frac{9}{3}-\frac{9}{2} = \frac{18}{6}-\frac{27}{6} = -\frac{9}{6} = -\frac{3}{2}$

#172) $\int_{-2}^{3}(t+2)(t-3)dt = \int_{-2}^{3} t^2-t-6\,dt = \frac{t^3}{3}-\frac{t^2}{2}-6t\Big|_{-2}^{3}$

$= \left(\frac{3^3}{3}-\frac{3^2}{2}-6(3)\right)-\left(\frac{(-2)^3}{3}-\frac{(-2)^2}{2}-6(-2)\right)$

$= \left(9-\frac{9}{2}-18\right)-\left(-\frac{8}{3}-\frac{4}{2}+12\right)$

$= -21-\frac{5}{2}+\frac{8}{3}$

$= -\frac{126}{6}-\frac{15}{6}+\frac{16}{6}$

$= \frac{-125}{6}$

#181) $\int_1^{16}\frac{dt}{t^{1/4}} = \int_1^{16} t^{-1/4}dt = \frac{t^{3/4}}{3/4}\Big|_1^{16} = \frac{4}{3}\left[16^{3/4}-1\right]$

#182) $\int_0^{2\pi}\cos(\theta)\,d\theta = \sin(\theta)\Big|_0^{2\pi} = \sin(2\pi)-\sin(0) = 0-0 = 0$

§1.5 #264

$\int (3x-2)^{-11}\,dx$; $u = 3x-2$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$= \frac{1}{3}\int u^{-11}\,du$

$= \frac{1}{3}\frac{u^{-10}}{-10} + C$

$= \frac{-1}{30}(3x-2)^{-10} + C$

#266

$\int \frac{x}{\sqrt{1-x^2}}\,dx$; $u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$

$= -\frac{1}{2}\int u^{-1/2}\,du$

$= -\cancel{\tfrac{1}{2}}\frac{u^{1/2}}{\cancel{1/2}} + C$

$= -\sqrt{1-x^2} + C$

#276

$\int \sin^7(\theta)\cos(\theta)\,d\theta$ $u = \sin(\theta)$

$du = \cos(\theta)\,d\theta$

$= \int u^7\,du = \frac{u^8}{8} + C$

$= \frac{\sin^8(\theta)}{8} + C$

$u = x^3 - 3$
$du = 3x^2 dx \quad \frac{1}{3} du = x^2 dx$

#28

$$\int \frac{x^2}{(x^3-3)^2} dx = \frac{1}{3} \int \frac{1}{u^2} du$$

$$= \frac{1}{3} \int u^{-2} du = \frac{1}{3} \cdot \frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{3} u^{-1} + C$$

$$= -\frac{1}{3} (x^3-3)^{-1} + C$$

#28

$u = 1 - x^2 \longrightarrow x^2 = 1 - u$
$du = -2x\,dx$
$-\frac{1}{2} du = x\,dx$

$$\int \frac{x^3}{\sqrt{1-x^2}} dx = \int \frac{x^2}{\sqrt{1-x^2}} x\,dx$$

$$= -\frac{1}{2} \int \frac{1-u}{\sqrt{u}} du$$

$$= -\frac{1}{2} \int \frac{1}{\sqrt{u}} - \sqrt{u}\, du = -\frac{1}{2} \int u^{-1/2} - u^{1/2} du$$

$$= -\frac{1}{2} \left[\frac{u^{1/2}}{1/2} - \frac{u^{3/2}}{3/2} \right] + C$$

$$= -\frac{1}{2} \left[2u^{1/2} - \frac{2}{3} u^{3/2} \right] + C$$

$$= -(1-x^2)^{1/2} - \frac{1}{3} (1-x^2)^{3/2} + C$$

$u = 5+t^2 \quad du = 2t \quad \frac{1}{2}du = t\,dt$

lower bound: $x=0 \rightarrow u = 5+0^2 = 5$

#294| $\displaystyle\int_0^2 \frac{t}{\sqrt{5+t^2}}\,dt$ upper bound: $x=2 \rightarrow u = 5+2^2 = 9$

$$= \frac{1}{2}\int_5^9 \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int_5^9 u^{-1/2}\,du = \cancel{\frac{1}{2}}\,\frac{u^{1/2}}{\cancel{1/2}}\Bigg|_5^9$$

$$= 9^{1/2} - 5^{1/2}$$

$$= 3 - \sqrt{5}$$

§1.6 #320| $\displaystyle\int e^{2x}dx$

$u = 2x$

$\frac{1}{2}du = dx$

$$= \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C$$

$$= \frac{1}{2}e^{2x} + C$$

#322| $\displaystyle\int 2^x dx = \int e^{\ln(2^x)}dx$

$u = x\ln(2)$

$\frac{1}{\ln(2)}du = dx$

$$= \int e^{x\ln(2)}\,dx$$

$$= \frac{1}{\ln(2)}\int e^u\,du$$

$$= \frac{1}{\ln(2)}e^u + C$$

$$= \frac{1}{\ln(2)}e^{x\ln(2)} + C$$

$$= \frac{1}{\ln(2)}2^x + C$$

$u = \ln(x)$
$du = \frac{1}{x}dx$

#330 $\int \frac{dx}{x\ln(x)} = \int \frac{du}{u}$

$= \ln(u) + C$
$= \ln(\ln(x)) + C$

#356 $\int_0^{\pi/4} \tan(x)\,dx = \int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx$ $\quad u = \cos x$, $-du = \sin x\,dx$

upper bd: $x = 0 \rightarrow u = \cos(0) = 1$
lower bd: $x = \pi/4 \rightarrow u = \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$= -\int_1^{\frac{\sqrt{2}}{2}} \frac{1}{u}\,du$

$= -\ln(u)\Big|_1^{\sqrt{2}/2}$

$= -\ln(\frac{\sqrt{2}}{2}) - (\underbrace{-\cancel{\ln(1)}}_{=0})$

$= -\ln(\frac{\sqrt{2}}{2})$

#360 $\int \frac{x}{x-100}\,dx$; $u = x - 100 \rightarrow u + 100 = x$
$du = dx$

$= \int \frac{u+100}{u}\,du = \int 1 + \frac{100}{u}\,du$

$= \int 1\,du + 100\int \frac{1}{u}\,du$

$= u + 100\ln(u) + C$
$= (x-100) + 100\ln(x-100) + C$