Section 5.5

#250 $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+3}$

$b_n = \frac{n}{n+3} \xrightarrow{n\to\infty} 1 \Rightarrow$ does not converge at all ("eventually oscillatory")

#252 $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n+3}}$

$b_n = \frac{1}{\sqrt{n+3}} \xrightarrow{n\to\infty} 0 \Rightarrow$ converges

Absolute?

no because $\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+3}} = 1$ and $\sum \frac{1}{\sqrt{n}}$ diverges (p-series with $p<1$)

$\Rightarrow$ conditional convergence

#254 $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{n!}$

$b_n = \frac{1}{n!} \xrightarrow{n\to\infty} 0 \Rightarrow$ converges

Absolute?

yes ~ $n! > 2^n \Rightarrow \frac{1}{n!} < \frac{1}{2^n}$, but $\sum \frac{1}{2^n}$ converges (geometric, $r = \frac{1}{2}$)

#261 $\sum_{n=1}^{\infty} (-1)^{n+1} \cos^2\left(\frac{1}{n}\right)$

$b_n = \cos^2\left(\frac{1}{n}\right) \xrightarrow{n\to\infty} \cos^2(0) = 1^2 = 1 \longrightarrow$ does not converge

#282 $R_N \le b_{N+1} \Rightarrow R_N \le \frac{1}{\sqrt{N+1}}$

Requiring $R_N \le \frac{1}{\sqrt{N+1}} < 10^{-3}$

$\Rightarrow \sqrt{N+1} > 10^3$

$\Rightarrow N+1 > 10^6$

$\Rightarrow N > 10^6 - 1$ ~ such N will guarantee the required bound on the remainder

§5.4 #318 $\sum_{n=1}^{\infty} \frac{10^n}{n!}$ Ratio test

$$\frac{\frac{10^{n+1}}{(n+1)!}}{\frac{10^n}{n!}} = \frac{10^{n+1}\, n!}{(n+1)!\, 10^n} = \frac{10}{n+1} \xrightarrow{n\to\infty} 0$$

$\Rightarrow$ Converges

#320 $\sum_{n=1}^{\infty} \frac{n^{10}}{2^n}$

$$\frac{\frac{(n+1)^{10}}{2^{n+1}}}{\frac{n^{10}}{2^n}} = \frac{(n+1)^{10}\, 2^n}{2^{n+1}\, n^{10}} = \frac{1}{2}\,\frac{(n^{10}+\ldots)}{n^{10}} \xrightarrow{n\to\infty} \frac{1}{2}$$

$\Rightarrow$ converges

#325 $\sum_{n=1}^{\infty} \frac{n!}{(n/e)^n} \rightsquigarrow$

$$\frac{\frac{(n+1)!}{\left(\frac{n+1}{e}\right)^{n+1}}}{\frac{n!}{\left(\frac{n}{e}\right)^n}} = \frac{(n+1)!\, e^{n+1}\, n^n}{(n+1)^{n+1}\, e^n\, n!} = \frac{\cancel{(n+1)}\, e}{\cancel{(n+1)}}\left(\frac{n}{n+1}\right)^n$$

But $\left(\frac{n}{n+1}\right)^n = e^{\overset{\to \infty \cdot 0}{n \ln\left(\frac{n}{n+1}\right)}} = e^{\frac{\ln\left(\frac{n}{n+1}\right)}{1/n}}$

$$\overset{L'H}{=} \exp\left(\lim_{n\to\infty} \frac{\frac{n+1}{n}\left(\frac{(n+1)-n}{(n+1)^2}\right)}{-1/n^2}\right)$$

$$= \exp(-1) < 1$$

$\Rightarrow$ Converges by ratio test

#328| $\sqrt[k]{a_k} = \sqrt[k]{\left(\frac{k-1}{2k+3}\right)^k} = \frac{k-1}{2k+3} \to \frac{1}{2}$

$\Rightarrow \sum a_k$ converges by root test

#330| $\sqrt[n]{|a_n|} = \sqrt[n]{\frac{(\ln(n))^{2n}}{n^n}} = \sqrt[n]{\left(\frac{(\ln(n))^2}{n}\right)^n}$

$= \frac{\ln(n)^2}{n} \xrightarrow{n\to\infty} 0$ (log << poly)

Thus $\sum a_n$ converges by root test

#332| note: we saw in class that $\sqrt[n]{n} \to 1$ as $n \to \infty$,

so

$\sqrt[n]{a_n} = \sqrt[n]{\frac{n}{e^n}} = \frac{\sqrt[n]{n}}{e} \xrightarrow{n\to\infty} \frac{1}{e} < 1$

$\Rightarrow \sum a_n$ converges by root test