Honors HW5 (due 4 March)

In HW3 you considered **surface area** of solids of revolution.

From the text, Section 2.4, p. 279, the surface area of a solid of revolution formed by rotating the curve $f(x)$ above $[a, b]$ around the x-axis is given by

$$\text{SurfaceArea} = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2}\mathrm{d}x.$$

In this project, you will look at "Gabriel's horn" which is a surface defined by rotating a curve around the x-axis. This surface is interesting because it has finite volume but infinite surface area (so you can fill up its volume with paint, but you cannot paint it itself).

1.) Use the washer method to find the volume of the surface bounded by the curves $\frac{1}{x}$, $y = 0$, and $x = 1$ (note: this will have an infinite domain). You should arrive at a finite volume.

2.) Use the surface area formula to **write down the integral** you would compute to find the surface area of Gabriel's horn.

3.) Plot the curves $y = \frac{1}{x}$ and $y = \frac{1}{x}\sqrt{1 + \frac{1}{x^4}}$ in Desmos (or your favorite plotting utility). Which curve is on "top" over the interval $[1, \infty)$?

4.) You should have found in 3.) that $\frac{1}{x}\sqrt{1 + \frac{1}{x^4}} > \frac{1}{x}$. Using this fact and the general idea that if $f(x) \geq g(x)$, then $\int f(x)\mathrm{d}x \geq \int g(x)\mathrm{d}x$ ("more area"), conclude that the integral for surface area is greater than an integral which results in ∞. This proves the surface area of Gabriel's horn is infinite.