Honors Homework 4 – MATH 2502 Spring 2019

The following problem concerns "Fourier polynomials" which were invented Joseph Fourier in 1807. These polynomials were used to discover the first analytical solution to the heat equation. They form the basis of Fourier series, which have become one of the most useful techniques in mathematics, applicable to electical engineering, acoustics, optics, signal processing, quantum mechanics, and many other fields of study.
Let f be a (continuous) function on the interval $[-\pi, \pi]$. Define the Fourier coefficients of f by the formulas

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)$$

and

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx).$$

For $N = 1, 2, 3, \ldots$ we define the Fourier polynomials by

$$P_N(x) = \frac{a_0}{2} + \sum_{k=1}^{N} [a_n \cos(nx) + b_n \sin(nx)].$$

Problems

1. Let $f(x) = x$. Find the Fourier coefficients a_0, a_1, a_2, a_3, a_4, and a_5. Also find b_0, b_1, b_2, b_3, b_4, and b_5. It is OK to use software to complete these calculations (by hand it's a lot of integrations by parts).

2. Use your answer above to write down the first four Fourier polynomials $P_1(x)$, $P_2(x)$, $P_3(x)$, $P_4(x)$, and $P_5(x)$.

3. Plot $y = x$ and also plot these polynomials on the interval $[-2\pi, 2\pi]$. (hint: easy to do on Desmos). What appears to be happening?

4. Repeat steps 1-4 but using the function $f(x) = x^2$. What appears to be happening?